NIMCET
Previous year paper 2012

Included Subjects
Mathematics
Logical Reasoning
Computer
English

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NIMCET 2012

Mathematics:

1. If H is the harmonic mean between P and Q, then $\frac{H}{P}+\frac{H}{Q}$ is
(A) 2 (B) $\frac{P+Q}{Q}$
(C) $\frac{PQ}{P+Q}$ (D) None of these

2. The number of values of k for which the system of equations $(k+1)x+8y=4k$ and $kx+(k+3)y=3k-1$ has infinitely many solutions, is
(A) 0 (B) 1
(C) 2 (D) infinite

3. The sum of ${}^{20}C_8+{}^{20}C_9+{}^{21}C_{10}+{}^{22}C_{11}-{}^{23}C_{11}$ is
(A) ${}^{22}C_{12}$ (B) ${}^{23}C_{12}$
(C) 0 (D) ${}^{21}C_{10}$

4. The value of $\cot^{-1}(21)+cot^{-1}(13)+cot^{-1}(-8)$ is
(A) 0 (B) π
(C) ∞ (D) $\frac{\pi}{2}$

5. Normal to the curve $y=x^3-3x+2$ at the point $(2,4)$ is
(A) $9x-y-14=0$ (B) $x-9y+40=0$
(C) $x+9y-38=0$ (D) $-9x+y+22=0$

6. The value of $\lim_{x\to\infty}\frac{\pi}{n}\left[\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\cdots+\sin\frac{(n-1)\pi}{n}\right]$ is
(A) 0 (B) π
(C) 2 (D) $\frac{\pi}{2}$

7. The point on the curve $y=6x-x^2$, where the tangent is parallel to X-axis is
(A) $(0,0)$ (B) $(2,8)$
(C) $(6,0)$ (D) $(3,9)$

8. If $I_1=\int_0^1 2^{x^2}dx$, $I_2=\int_0^1 2^{x^3}dx$, $I_3=\int_1^2 2^{x^2}dx$ and $I_4=\int_1^2 2^{x^3}dx$, then
(A) $I_1=I_2$ (B) $I_2>I_1$
(C) $I_3>I_4$ (D) $I_4>I_3$

9. The value of Integral $\int_0^{\pi/2}\log\tan x\,dx$ is
(A) π (B) $\frac{\pi}{2}$
(C) $\frac{\pi}{3}$ (D) 0

10. A determinant is chosen at random from the set of all determinants of matrices of order 2 with elements 0 and 1 only. The probability that the determinant chosen is non-zero, is
(A) $\frac{3}{16}$ (B) $\frac{3}{8}$
(C) $\frac{1}{4}$ (D) None of these

11. If $\sin^2x=1-\sin x$, then $\cos^4x+cos^2x$ is equal to
(A) 0 (B) 1
(C) $\frac{2}{3}$ (D) -1

12. The equation of the plane passing through the point $(1,2,3)$ and having the vector $N=3i-j+2k$ as its normal, is
(A) $2x-y+3z+7=0$ (B) $3x-y+2z+7=0$
(C) $3x-y+2z=7$ (D) $3x+y+2z=7$

13. The value of $\int_0^{\sin^2x}\sin^{-1}\sqrt{t}\,dt+\int_0^{\cos^2x}\cos^{-1}\sqrt{t}\,dt$ is
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$
(C) 1 (D) None of these

14. Coefficients of quadratic equation $ax^2+bx+c=0$ are chosen by tossing three fair coins, where 'head' means one and 'tail' means two. Then the probability that roots of the equation are imaginary, is
(A) $\frac{7}{8}$ (B) $\frac{5}{8}$
(C) $\frac{3}{8}$ (D) $\frac{1}{8}$

15. In a class of 100 students, 55 students have passed in Mathematics and 67 students have passed in Physics. Then, the number of students who have passed in Physics only, is

(A) 22 (B) 33
(C) 10 (D) 45

16. If $(4,-3)$ and $(-9,7)$ are the two vertices of a triangle and $(1,4)$ is its centroid, then the area of triangle is
(A) $\frac{138}{2}$ (B) $\frac{319}{2}$
(C) $\frac{183}{2}$ (D) $\frac{381}{2}$

17. The equation of the ellipse with major axis along the x-axis and passing through the points $(4,3)$ and $(-1,4)$ is
(A) $15x^2+7y^2=247$ (B) $7x^2+15y^2=247$
(C) $16x^2+9y^2=247$ (D) $9x^2+16y^2=247$

18. If the circles $x^2+y^2+2x+2ky+6=0$ and $x^2+y^2+2ky+k=0$ intersect orthogonally, then k is
(A) 2 or $-\frac{3}{2}$ (B) -2 or $-\frac{3}{2}$
(C) 2 or $\frac{3}{2}$ (D) -2 or $\frac{3}{2}$

19. Focus of the parabola $x^2+y^2-2xy-4(x+y-1)=0$ is
(A) $(1,1)$ (B) $(1,2)$
(C) $(2,1)$ (D) $(0,2)$

20. If a, b and c are unit vectors such that $a+b+c=0$, then the value of $a.b+b.c+c.a$ is
(A) $\frac{2}{3}$ (B) $-\frac{2}{3}$
(C) $\frac{3}{2}$ (D) $\frac{-3}{2}$

21. If two towers of heights h_1 and h_2 subtend angles 60° and 30° respectively at the mid-point of the line joining their feet, then $h_1:h_2$ is
(A) 1 : 2 (B) 1 : 3
(C) 2 : 1 (D) 3 : 1

22. If the vectors $a=(1,x,-2)$ and $b=(x,3,-4)$ are mutually perpendicular, then the value of x is
(A) -2 (B) 2
(C) 4 (D) -4

23. What is the value of a for which $f(x)=\begin{cases}\sin x, & if\ x\le\frac{\pi}{2}\\ ax, & if\ x>\frac{\pi}{2}\end{cases}$ is continuous?
(A) π (B) $\frac{\pi}{2}$
(C) $\frac{2}{\pi}$ (D) 0

24. If the real number x when added to its inverse gives the minimum value of the sum, then the value of x is equal to
(A) -2 (B) 2
(C) 1 (D) -1

25. If $\cos(\alpha+\beta)=\frac{4}{5}$ and $\sin(\alpha-\beta)=\frac{5}{13}, 0<\alpha,\beta<\frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to
(A) $\frac{56}{33}$ (B) $\frac{63}{65}$
(C) $\frac{16}{63}$ (D) $\frac{33}{56}$

26. The number of words that can be formed by using the letters of the word 'MATHEMATICS' that start as well as end with T is
(A) 80720 (B) 90720
(C) 20860 (D) 37528

27. If $A-B=\frac{\pi}{4}$, then $(1+\tan A)(1-\tan B)$ is equal to
(A) 2 (B) 1
(C) 0 (D) 3

28. Let $P(E)$ denote the probability of event E. Given $P(A)=1, P(B)=\frac{1}{2}$, the values of $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$ respectively are
(A) $\frac{1}{4},\frac{1}{2}$ (B) $\frac{1}{2},\frac{1}{4}$
(C) $\frac{1}{2},1$ (D) $1,\frac{1}{2}$

29. The number of different license plates that can be formed in the format 3 English letters (A ..... Z) followed by 4 digits (0, 1, ... , 9) with repetitions allowed in letters and digits is equal to
(A) $26^3\times10^4$ (B) 26^3+10^4
(C) 36 (D) 26^3

30. Which of the following is correct?
(A) $\sin 1°>\sin 1$ (B) $\sin 1°<\sin 1$
(C) $\sin 1°=\sin 1$ (D) $\sin 1°=\frac{\pi}{180}\sin 1$

31. If a,b,c are non-coplanar vectors and λ is a real number, then the vectors $a+2b+3c, \lambda b+4c$ and $(2\lambda-1)c$ are non-coplanar for

(A) all values of λ (B) all except one value of λ
(C) all except two values of λ (D) no value of λ

32. Suppose values taken by a random variable X are such that $a \leq x_i \leq b$, where x_i denotes the value of X in the ith case for $i = 1, 2, 3, \ldots, n$, then

(A) $(b-a)^2 \geq \text{Var}(X)$ (B) $\frac{a^2}{4} \leq \text{Var}(X)$
(C) $a^2 \leq \text{Var}(X) \leq b^2$ (D) $a \leq Var(X) \leq b$

33. If ω is the cube root of unity, then the system of equations $x + \omega^2 y + \omega z = 0, \omega x + y + \omega^2 z = 0$ and $\omega^2 x + \omega y + z = 0$ is
(A) consistent and has unique solution
(B) consistent and has more than one solution
(C) inconsistent
(D) None of the above

34. If $x = \log_a bc$, $y = \log_b ca$ and $z = \log_c ab$, then $\frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z}$ is equal to
(A) abc (B) $\sqrt{ab} + \sqrt{bc} + \sqrt{ca}$
(C) 1 (B) $x + y + z$

35. If $2^a = 3^b = 6^{-c}$, then $ab + bc + ca$ is equal to
(A) 1 (B) 2
(C) 0 (D) None of these

36. If e and e' be the eccentricities of a hyperbola and its conjugate, then $\frac{1}{e^2} + \frac{1}{e'^2}$ is equal to
(A) 0 (B) 1
(C) 2 (D) None of these

37. If a fair coin is tossed n times, then the probability that the head comes odd number of times is
(A) $\frac{1}{2}$ (B) $\frac{1}{2^n}$
(C) $\frac{1}{2^{n-1}}$ (D) None of these

38. If $\sin(\pi \cos\theta) = \cos(\pi \sin\theta)$, then $\sin 2\theta$ is equal to
(A) $\pm\frac{3}{4}$ (B) $\pm\frac{1}{3}$
(C) $\pm\frac{1}{4}$ (D) $\pm\frac{4}{3}$

39. In which of the following regular polygons, the number of diagonals is equal to number of sides?
(A) Pentagon (B) Square
(C) Octagon (D) Hexagon

40. One hundred identical coins each with probability P of showing up heads are tossed. If $0 < P < 1$ and the probability of heads showing on 50 coins is equal to that of heads on 51 coins, then the value of P is
(A) $\frac{1}{2}$ (B) $\frac{49}{101}$
(C) $\frac{50}{101}$ (D) $\frac{51}{101}$

41. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$, where x is a variable has real roots. Then, the interval of p is
(A) $(0, 2\pi)$ (B) $(-\pi, 0)$
(C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (D) $(0, \pi)$

42. Number of real roots of $3x^5 + 15x - 8 = 0$ is
(A) 3 (B) 5
(C) 1 (D) 0

43. The value of k for which the set of equations $3x + ky - 2z = 0, x + ky + 3z = 0$ and $2x + 3y - 4z = 0$ has a non-trivial solution, is
(A) $\frac{15}{2}$ (B) $\frac{17}{2}$
(C) $\frac{31}{2}$ (D) $\frac{33}{2}$

44. If $x = \log_3 5$, $y = \log_{17} 25$, then which one of the following is correct?
(A) $x > y$ (B) $x < y$
(C) $x \leq y$ (D) $x = y$

45. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then A^n for any natural number n is
(A) $\begin{bmatrix} n & n \\ 0 & n \end{bmatrix}$ (B) $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (D) None of these

46. A problem in Mathematics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$, respectively. If they all try to solve the problem, what is the probability that the problem will be solved?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$
(C) $\frac{1}{3}$ (D) $\frac{3}{4}$

47. The function x^x decreases in the interval
(A) $(0, e)$ (B) $(0, 1)$
(C) $\left(0, \frac{1}{e}\right)$ (D) None of these

48. If $a + b + c = 0, |a| = 3, |b| = 5, |c| = 7$, then angle between the vectors a and b is
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

49. If $\theta (0 \leq \theta \leq \pi)$ is the angle between the vectors a and b, then $\frac{|a \times b|}{a.b}$ equals to
(A) $-\cot\theta$ (B) $\tan\theta$
(C) $-\tan\theta$ (D) $\cot\theta$

50. If $f(a + b) = f(a) \times f(b)$ for all a and b and $f(5) = 2, f'(0) = 3$, then $f'(5)$ is equal to
(A) 2 (B) 4
(C) 6 (D) 8

Reasoning/Aptitude:

51. If a man walks at the rate of 4 km/h, he misses a train by only 6 min. However, if he walks at the rate of 5 km/h he reaches the station 6 min before the arrival of the train. The distance covered by him to reach the station is
(A) 4 km (B) 7 km
(C) 9 km (D) 5 km

52. The missing number in the given series 3, 6, 6, 12, 9, ... , 12 is
(A) 15 (B) 18
(C) 11 (D) 13

53. A man runs 20 m towards east and turns right, runs 10 m and turns right, runs 9 m and turns left, runs 5 m and turns left, runs 12 m and finally turns left and runs 6 m. Which direction is the man facing?
(A) North (B) South
(C) East (D) West

54. In a club, there are certain number of males and females. If 15 females are absent, then number of males will be half of females. If 45 males are absent, then female strength will be 5 times that of males. Number of males actually present is
(A) 45 (B) 80
(C) 105 (D) 175

55. The missing number in the following series 6, 12, 21, ... , 48 is
(A) 40 (B) 33
(C) 38 (D) 45

Directions (Q. Nos. 56-58) Read the following passage carefully and answer the questions.

Six boys A, B, C, D, E and F are marching in a line. They are arranged according to their heights, the tallest being at the back and the shortest in the front. F is between B and A. E is shorter than D but taller than C who is taller than A. E and F have two boys between them. A is not the shortest among them.

56. Where is E?
(A) Between A and B (B) Between C and A
(C) Between D and C (D) In front of C

57. If we start counting from the shortest, which boy is fourth in the line?
(A) E (B) A
(C) D (D) C

58. Who is next to the shortest?
(A) C (B) B
(C) E (D) F

59. Let x, y and z be distinct integers. x and y are odd and positive and z is even and positive. Which one of the following statements cannot be true?
(A) $(x - z)^2 y$ is even (B) $(x - z)y^2$ is odd
(C) $(x - z)y$ is odd (D) $(x - y)^2 z$ is even

60. Pointing to a man in the photograph a lady said, "The father of his brother is the only son of my mother." How is this man in photograph related to the lady?

(A) Brother (B) Son
(C) Grandson (D) Nephew

61. Find the odd number in the following series.
2, 9, 28, 65, 126, 216, 344, ...
(A) 28 (B) 65
(C) 126 (D) 216

62. Average age of students of an adult school 40 yr. 120 new students whose average age is 32 yr joined the school. As a result the average age is decreased by 4 yr. The number of students of the school after joining of the new students is
(A) 1200 (B) 120
(C) 360 (D) 240

63. The letters P, Q, R, S, T, U and V not necessarily in that order represent seven consecutive integers from 22 to 33 and
1. U is as much less than Q as R is greater than S.
2. V is greater than U.
3. Q is the middle term.
4. P is greater than S.

Then, the sequence of letters from the lowest value to the highest value, is
(A) TVPQRSU (B) TRSQUPV
(C) TUSQRPV (D) TVPQSRU

64. The minimum number of tiles of size 16 by 24 required to form a square by placing them adjacent to one another is
(A) 6 (B) 8
(C) 11 (D) 16

65. Five persons K, L, M, N and O are sitting around a dining table. K is the mother of M, M is actually the wife of O, N is the brother of K and L is the husband of K. How is N related to L?
(A) Son (B) Cousin
(C) Brother (D) Brother-in-law

66. Three men A, B and C play cards. If one loses the game he has to give ₹ 3. If he wins the game he will gain ₹ 3 each from the other two losers. If A has won 3 games, B loses ₹ 3, C wins ₹12, then the total number of games played is
(A) 12 (B) 21
(C) 20 (D) 6

Directions (Q. Nos. 67-69) Read the following passage carefully and answer the questions.

- A causes B or C but not both.
- F occurs only if B occurs.
- D occurs, if B or C occurs.
- E occurs only if C occurs.
- J occurs only if E or F occurs.
- D causes G or H or both.
- H occurs, if E occurs.
- G occurs, if F occurs.

67. If A occurs, which may occur?
I. F and G II. E and H III. D
(A) Only I
(B) Only II
(C) I and III or II and III, but not both
(D) I, II and III

68. If B occurs, which must occur?
(A) D (B) G
(C) H (D) J

69. If J occurs, which must have occurred?
(A) Both E and F (B) Either B or C
(C) Both B and C (D) None of these

70. If 'ROAST' is coded as 'PQYUR' in a certain language, then 'SLOPPY' is coded in that language as
(A) MRNAQN (B) NRMNQA
(C) QNMRNA (D) RAANNMQ

71. If 'lelibroon' means 'yellow hat', 'plekafroti' means 'flower garden' and 'frotimix' means 'garden salad', then which word could mean 'yellow flower'?
(A) lelifroti (B) lelipleka
(C) plekabroon (D) frotibroon

72. If + is $*$, − is / and / is −, then
$6 - 9 + 8 * 3/20$ is equal to
(A) −2 (B) 6
(C) 10 (D) 12

73. In a certain year, there were exactly four Fridays and four Mondays in January. On what day of the week did the 20th of January fall that year?
(A) Saturday (B) Sunday
(C) Thursday (D) Tuesday

74. Krishna said, "This girl is the wife of grandson of my mother". How is Krishna related to girl?

(A) Father (B) Father-in-law
(C) Husband (D) Grandfather

75. Instead of walking along two adjacent sides of a rectangular field, a boy took a shortcut along the diagonal of the field and saved a distance equal to half the longer side. The ratio of the shorter side of the rectangle to the longer side is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$
(C) $\frac{1}{4}$ (D) $\frac{3}{4}$

76. Each word in parenthesis below is formed in a method. This method is used in all four examples.

$SNIP\ (NICE)\ PACE$

$TEAR\ (EAST)\ FAST$

$TRAY\ (RARE)\ FIRE$

$POUT\ (OURS)\ CARS$

Based on this method, the word in the parenthesis of $CANE(?)\ BATS$ is

(A) *NEAT* (B) *CATS*
(C) *ANTS* (D) *NETS*

77. A study of native born residents in an area of Adivasis found that two-third of the children developed considerable levels of nearsightedness after starting school, while their illiterate parents and grandparents, who had no opportunity for formal schooling, showed no signs of this disability. If the above statements are true, which of the following conclusions is most strongly supported by them?

(A) Only people who have the opportunity for formal schooling develop nearsightedness

(B) People who are illiterate do not suffer from nearsightedness

(C) The nearsightedness in the children is caused by the visual stress required by reading and other class work

(D) Only literate people are nearsighted

Directions (Q. Nos. 78-80) Read the following passage carefully and answer the questions.

Five roommates Randy, Sally, Terry, Uma and Vernon each do one housekeeping task mopping, sweeping, laundry, vacuuming or dusting one day a week, Monday through Friday.

- Vernon does not vacuum and does not do his task on Tuesday.
- Sally does the dusting and does not do it on Monday of Friday.
- The Mopping is done on Thursday.
- Terry does his task, which is not vacuuming on Wednesday.
- The laundry is done on Friday and not by Uma.
- Randy does his task on Monday.

78. The task done by Terry on Wednesday is

(A) vacuuming (B) dusting
(C) mopping (D) sweeping

79. The day on which the vacuuming is done, is

(A) Friday (B) Monday
(C) Tuesday (D) Wednesday

80. Sally does dusting on

(A) Friday (B) Monday
(C) Tuesday (D) Wednesday

Directions (Q. Nos. 81-82) Read the following passage carefully and answer the questions.

P, Q, R, S, T, U, V and W are sitting round the circle and are facing the centre. P is second to the right of T, T is the neighbour of R and V. S is not the neighbour of P, V is the neighbour of U, Q is not between S and W and W is not between U and S.

81. Which two of following are not neighbours?

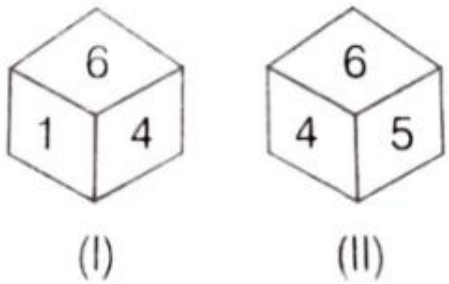
(A) RV (B) UV
(C) RP (D) QW

82. What is the position of S?

(A) Between U and V
(B) Second to the right of P
(C) To the immediate right of W
(D) Data inadequate

83. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit in the unit's place is 3 more than the digit in ten's place, then the number is
(A) 24 (B) 63
(C) 36 (D) 42

84. Two positions of a dice are shown below. When number 1 is on the top, what number will be at the bottom?



(A) 2 (B) 3
(C) 5 (D) Cannot be determined

85. A, B, C, D, E, F and G are sitting in a line facing East. C is immediate to the right of D. B is at one of the extreme ends and has E as his neighbour. G is between E and F. D is sitting third from the South end. What is sitting third from North?
(A) A (B) E
(C) F (D) G

86. There is a family party consisting of two fathers, two mothers, two sons, one father-in-law, one mother-in-law, one daughter-in-law, one grandfather, one grandmother and one grandson.

What is the minimum number of persons required, so that is possible?
(A) 5 (B) 6
(C) 7 (D) 8

87. If A is brother of B, C is brother of B and A is brother of D, then which of the following must be true?
(A) A is brother of C (B) B is brother of C
(C) D is brother of C (D) B is brother of D

Directions (Q. Nos. 88-90) Read the following passage carefully and answer the questions.

Five houses lettered A, B, C, D and E are built in a row next to each other. The houses are lined up in the order A, B, C, D and E. Each of the five houses have coloured roofs and chimneys. The roof and chimney of each house must be painted as follows.

1. The roof must be painted either green, red or yellow.
2. The chimney must be painted either white, black or red.
3. No house may have the same colour chimney as the colour of roof.
4. No house may use any of the same colours that adjacent house uses.
5. House E has a green roof.
6. House B has a red roof and a black chimney.

88. Which of the following is true?
(A) Atleast two houses have black chimney
(B) Atleast two houses have red roofs
(C) Atleast two houses have white chimneys
(D) Atleast two houses have green roofs

89. If house C has a yellow roof, then which of the following must be true?
(A) House E has a white chimney
(B) House E has a black chimney
(C) House E has a red chimney
(D) House D has a red chimney

90. What is the maximum number of green roofs?
(A) 1 (B) 2
(C) 3 (D) 4

English:

91. For a word, four spellings are given. Choose the correct one.
(A) Cieling (B) Cealing
(C) Ceiling (D) Ceeling

92. Choose the wrongly spelt word.
(A) Believe (B) Relieve
(C) Grieve (D) Decieve

93. Choose the word or phrase that is most similar in the meaning to the word POLEMIC.
(A) Black (B) Magnetic
(C) Grimace (D) Controversial

94. The sentence below has 2 blanks. Fill in the blanks picking the appropriate pair of words from the ones given below that best completes the meaning of the sentences.
The most technologically advanced societies have been responsible for the greatest ………….; indeed, savagery seems to be in direct proportion to ………..
(A) wars; viciousness (B) catastrophes: ill-will
(C) atrocities; development (D) triumphs; civilisation

95. Fill in the blank with the correct form of tense.
The thief …… before the police came
(A) escaped (B) had escaped
(C) will escape (D) had been escaped

96. Fill in the blank with appropriate words given.
Anne had to pay for everything because as usual, Peter ….. his wallet at home.
(A) had left (B) was leaving
(C) left (D) leave

97. Pick the synonym of the word 'Meagre'
(A) Helpful (B) Abundant
(C) Essential (D) Limited

98. Choose the words that best express the meaning of the given idiom-Mud slinging.
(A) Giving pain
(B) Abusing someone
(C) Laying blame
(D) Damaging the reputation

99. Pick the antonym of the word 'Timid'.
(A) Bold (B) Lazy
(C) Calm (D) Slow

100. Pick the part of the sentence that has an error.
If you would have come to me, I would have helped you.
(A) If you would have (B) Come to me
(C) I would have (D) Helped you

101. Choose the word or phrase that is most nearly opposite in meaning to the word 'Extrinsic'.
(A) Reputable (B) Inherent
(C) Ambitious (D) Cursory

102. Select the alternative giving the closest meaning of the idiom – To eat a humble pie.
(A) To become a vegetarian
(B) Disinfecting everything
(C) To fill one's belly
(D) To say you are sorry for a mistake that you made

103. Pick the antonym of the word 'Fabricate'.
(A) Construct (B) Weaken
(C) Dismantle (D) Evolve

Directions (Q. Nos. 104-110) *Fill in the blank with correct option to make a proper sentence.*

104. The people …. you socialize are called friends.
(A) with whom (B) who
(C) with who (D) whom

105. …… to school yesterday?
(A) Did you walk (B) Did you walked
(C) Do you walk (D) Have you walked

106. There was no ……. in the railway compartment for additional passengers.
(A) space (B) place
(C) seat (D) room

107. And now for this evening's main headline; Britain …. Another Olympic gold medal.
(A) had won (B) wins
(C) won (D) has won

108. If she ……. About his financial situation, she would have helped him out.
(A) knew (B) had been knowing
(C) had known (D) have known

109. I am sure she can teach computers as well. She's not ……….. new to the subject.
(A) all together (B) altogether
(C) alltogether (D) together

110. You are trying to drag me …… a controversy.
(A) in (B) into
(C) from (D) for

English:

111. An I/O processor controls the flow of information between
(A) cache memory and I/O devices
(B) main memory and I/O devices
(C) two I/O devices
(D) cache and main memories

112. Which of following devices will take highest time in taking the backup of the data from a computer?
(A) Magnetic disk (B) Pen drive
(C) CD (D) Magnetic tape

113. ROM is a kind of
(A) primary memory (B) cache memory
(C) removable memory (D) secondary memory

114. The errors that can be pointed out by compilers are
(A) syntax errors (B) semantic errors
(C) logical errors (D) internal errors

115. Let $x = 11111010$ and $y = 00001010$ be two 8-bit 2's complement numbers. Their product in 2's complement notation is
(A) 11000100 (B) 10011100
(C) 10100101 (D) 11010101

116. The range of numbers that can be stored in 8 bits, if negative numbers are stored in 2's complement form is
(A) -128 to $+128$ (B) -127 to $+128$
(C) -128 to $+127$ (D) -127 to $+127$

117. Primary storage is …… as compared to secondary memory.
(A) slow and expensive (B) fast and inexpensive
(C) fast and expensive (D) slow and inexpensive

118. Which of the following units is used to supervise each instruction in the CPU?
(A) Control unit (B) Accumulator
(C) ALU (D) Control Register

119. $(2FAOC)_{16}$ is equivalent to
(A) $(195\ 084)_{10}$
(B) $(00101111101000001100)_2$
(C) Both (A) & (B)
(D) None of the above

120. The decimal equivalent of octal number 111010
(A) 81 (B) 72
(C) 71 (D) 61

Answer Key

1. A	**13.** A	**25.** A	**37.** A	**49.** C	**61.** D	**73.** B	**85.** D	**97.** D	**109.** B
2. B	**14.** A	**26.** B	**38.** A	**50.** C	**62.** D	**74.** B	**86.** A	**98.** D	**110.** A
3. C	**15.** D	**27.** A	**39.** A	**51.** A	**63.** C	**75.** D	**87.** A	**99.** A	**111.** B
4. B	**16.** C	**28.** D	**40.** D	**52.** B	**64.** A	**76.** C	**88.** B	**100.** A	**112.** D
5. C	**17.** B	**29.** A	**41.** D	**53.** A	**65.** D	**77.** C	**89.** A	**101.** B	**113.** D
6. A	**18.** A	**30.** B	**42.** C	**54.** B	**66.** A	**78.** D	**90.** C	**102.** D	**114.** A
7. D	**19.** A	**31.** C	**43.** D	**55.** B	**67.** C	**79.** B	**91.** C	**103.** A	**115.** A
8. D	**20.** D	**32.** A	**44.** A	**56.** C	**68.** B	**80.** C	**92.** D	**104.** A	**116.** B
9. D	**21.** B	**33.** B	**45.** B	**57.** D	**69.** B	**81.** A	**93.** D	**105.** A	**117.** C
10. B	**22.** A	**34.** C	**46.** D	**58.** D	**70.** C	**82.** D	**94.** C	**106.** D	**118.** A
11. B	**23.** C	**35.** C	**47.** C	**59.** A	**71.** B	**83.** C	**95.** B	**107.** B	**119.** C
12. C	**24.** B	**36.** B	**48.** B	**60.** D	**72.** C	**84.** C	**96.** C	**108.** D	**120.** B

Solution

1. (a) Given that, H is the harmonic mean between P and Q.

i.e., $H = \frac{2PQ}{P+Q} \Rightarrow \frac{H}{2} = \frac{PQ}{P+Q}$

$\Rightarrow \frac{2}{H} = \frac{P+Q}{PQ}$ ...(i)

Now, $\frac{H}{P} + \frac{H}{Q} = H\left(\frac{P+Q}{PQ}\right) = H \cdot \frac{2}{H} = 2$ [from Eq. (i)]

2. (b) Given system of equations,

$(k+1)x + 8y = 4k$

$kx + (k+3)y = 3k - 1$

Since, the given system has infinitely many solutions

$\therefore \frac{k+1}{k} = \frac{8}{k+3} = \frac{4k}{3k-1}$

Taking Ist and IIIrd part,

$(k+1)(3k-1) = 4k^2$

$\Rightarrow 3k^2 + 2k - 1 = 4k^2$

$\Rightarrow k^2 - 2k + 1 = 0$

$\Rightarrow (k-1)^2 = 0$

$\therefore k = 1$

3. (c) $({}^{20}C_8 + {}^{20}C_9) + {}^{21}C_{10} + {}^{22}C_{11} - {}^{23}C_{11}$

$= ({}^{21}C_9 + {}^{21}C_{10}) + {}^{22}C_{11} - {}^{23}C_{11}$

$(\because {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$

$= ({}^{22}C_{10} + {}^{22}C_{11}) - {}^{23}C_{11} = {}^{23}C_{11} - {}^{23}C_{11}$

$= 0$

4. (b) $\cot^{-1}(21) + \cot^{-1}(13) + \cot^{-1}(-8)$

$\Rightarrow \tan^{-1}\left(\frac{1}{21}\right) + \tan^{-1}\left(\frac{1}{13}\right) + \cot^{-1}(-8)$

$\left(\because \cot^{-1}x = \tan^{-1}\frac{1}{x}\right)$

$\Rightarrow \tan^{-1}\left\{\frac{\frac{1}{21}+\frac{1}{13}}{1-\frac{1}{21}\cdot\frac{1}{13}}\right\} + \cot^{-1}(-8)$

$\left\{\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$

$\Rightarrow \tan^{-1}\left(\frac{34}{272}\right) + \tan^{-1}\left(-\frac{1}{8}\right) = \tan^{-1}\left(\frac{34}{272}\right) + \pi - \tan^{-1}\left(\frac{1}{8}\right)$

$\Rightarrow \pi + \tan^{-1}\left\{\frac{\frac{34}{272}-\frac{1}{8}}{1+\frac{34}{272}\cdot\frac{1}{8}}\right\} = \tan^{-1}\left\{\frac{34-34}{2210}\right\} + \pi$

$= \pi + \tan^{-1}(0) = 0 + \pi = \pi$

5. (c) Given curve, $y = x^3 - 3x + 2$

Now, $\frac{dy}{dx} = 3x^2 - 3$

$\Rightarrow \frac{dy}{dx}_{\text{at }(2,4)} = 3(2)^2 - 3 = 12 - 3 = 9$

$\therefore$ Slope of normal $= -\frac{1}{9}$

Hence, the equation of normal at point (2, 4)

$\Rightarrow (y-4) = -\frac{1}{9}(x-2)$

$\Rightarrow 9y - 36 = -x + 2$

$\Rightarrow x + 9y = 38$

$\Rightarrow x + 9y - 38 = 0$

6. (a) $\lim_{n\to\infty} \frac{\pi}{n}\left\{\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \ldots + \sin\left(\frac{n-1}{n}\right)\pi\right\}$

$= \lim_{n\to\infty} \frac{\pi}{n}\left\{\sin\left(\frac{\pi}{n}\right) + \sin\left(\frac{\pi}{n}+\frac{\pi}{n}\right) + \sin\left(\frac{\pi}{n}+\frac{2\pi}{n}\right) + \ldots + \sin\left(\frac{\pi}{n}+\frac{n\pi}{n}\right)\right\}$

$\because \sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \ldots + \sin(\alpha+n\beta) = \frac{\sin\left(\frac{2\alpha+n\beta}{2}\right)\cdot\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}$

$= \lim_{n\to\infty} \frac{\pi}{n} \cdot \frac{\sin\left\{\frac{\pi}{n}+\left(\frac{\pi}{n}+\frac{n\pi}{n}\right)\right\}\cdot\sin\frac{n}{2}\cdot\frac{\pi}{n}}{\sin\frac{\pi}{2n}}$

$= \lim_{n\to\infty} \frac{\pi}{n} \cdot \frac{\sin\left(\frac{2\pi+n\pi}{n}\right)\cdot\sin\frac{\pi}{2}}{\sin\frac{\pi}{2n}}$

$= \lim_{n\to\infty} \frac{1}{2\left(\frac{\sin\frac{\pi}{2n}}{\frac{\pi}{2n}}\right)} \cdot \sin\left(\pi + \frac{2\pi}{n}\right)\cdot 1 \quad \left(\because \lim_{\theta\to\infty}\frac{\sin\frac{1}{\theta}}{\frac{1}{\theta}} = 1\right)$

$$= \frac{1}{2 \cdot 1} \cdot \sin(\pi + 0)$$
$$= \frac{1}{2} \cdot 0 = 0$$

7. (d) Given curve, $y = 6x - x^2$ ...(i)

On differentiating w.r.t x,
$$\frac{dy}{dx} = 6 - 2x$$
$\because$ Slope of tangent parallel to x-axis is $\frac{dy}{dx} = 0$
$$\therefore \quad 6 - 2x = 0 \Rightarrow x = 3 \quad \text{[from Eq. (i)]}$$
$$y = 6(3) - (3)^2 = 18 - 9$$
$$y = 9$$
$\therefore$ Only one point (3, 9) at which the tangent is parallel to x-axis.

8. (d) $\because \quad x^2 > x^3 \quad \forall x \in (0, 1)$
$$\Rightarrow \quad 2^{x^2} > 2^{x^3} \forall x \in (0,1)$$
$$\Rightarrow \quad \int_0^1 2^{x^2} dx > \int_0^1 2^{x^3} dx$$
$$\Rightarrow \quad I_1 > I_2$$
Now, $\quad x^2 < x^3, \quad \forall x \in (1,2)$
$$\Rightarrow \quad 2^{x^2} < 2^{x^3}, \forall x \in (1,2)$$
$$\Rightarrow \quad \int_1^2 2^{x^2} dx < \int_1^2 2^{x^3} dx$$
$$\Rightarrow \quad I_3 < I_4 \quad \text{or} \quad I_4 > I_3$$

9. (d) Let $I = \int_0^{\pi/2} \log \tan x \, dx$ ...(i)

Use definite integeral property,
$$I = \int_0^{\pi/2} \log \tan\left(\frac{\pi}{2} - x\right) dx$$
$$= \int_0^{\pi/2} \log \cot x \, dx \quad \text{...(ii)}$$
On adding Eqs. (i) and (ii),
$$2I = \int_0^{\pi/2} (\log \tan x + \log \cot x) \, dx$$
$$(\because \log m + \log n = \log mn)$$
$$= \int_0^{\pi/2} \log(\tan x \cdot \cot x) \, dx$$
$$= \int_0^{\pi/2} \log 1 dx = \int_0^{\pi/2} 0 \, dx$$
$$= 0$$

10. (b) The total sample events $n(s) = 4 \cdot (2)^2 = 4 \times 4 = 16$ and total favourable cases $n(E) = 6$

which is $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{6}{16} = \frac{3}{8}$

11. (b) Given $\sin^2 x = 1 - \sin x$
$$\Rightarrow \quad 1 - \cos^2 x = 1 - \sin x$$
$$\Rightarrow \quad \sin x = \cos^2 x \quad \text{...(i)}$$
Now, $\cos^4 x + \cos^2 x = (\cos^2 x)^2 + \cos^2 x$
$$= (\sin x)^2 + \sin x$$
$$= \sin^2 x + \sin x$$
$$= (1 - \sin x) + \sin x \quad \text{[from Eq. (i)]}$$
$$= 1$$

12. (c) The equation of the plane passing through the point (1, 2, 3) and having the vector $\mathbf{N} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ as its normal is
$$3(x - 1) - 1(y - 2) + 2(z - 3) = 0$$
$$\Rightarrow \quad 3x - y + 2z + (-3 + 2 - 6) = 0$$
$$\Rightarrow \quad 3x - y + 2z = 7$$

13. (a) Let $f(z) = \int_0^{\sin^2 x} \sin^{-1}\sqrt{t} \, dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t} \, dt$

Differentiating on both sides by Leibnitz rule,
$$f'(x) = \sin^{-1}(\sin x)(2 \sin x \cos x) + \cos^{-1}(\cos x)(-2 \sin x \cdot \cos x)$$
$$= x \cdot \sin 2x - x \cdot \sin 2x$$
$$= 0$$
$$\Rightarrow \quad f(x) = \text{Constant}$$
Now, we check the constant value of this integration on different value of x.

(i) At $\left(x = \frac{\pi}{4}\right)$,
$$f\left(\frac{\pi}{4}\right) = \int_0^{1/2} \sin^{-1}\sqrt{t} \, dt + \int_0^{1/2} \cos^{-1}\sqrt{t} \, dt$$
$$= \int_0^{1/2} (\sin^{-1}\sqrt{t} + \cos^{-1}\sqrt{t}) \, dt = \int_0^{1/2} \frac{\pi}{2} dt$$
$$= \frac{\pi}{2}\left(\frac{1}{2} - 0\right) = \frac{\pi}{4}$$

(ii) At $(x = 0)$,
$$f(0) = 0 + \int_0^1 \cos^{-1}\sqrt{t} \, dt$$
Let $\quad t = \cos^2\theta, \quad dt = -\sin 2\theta \, d\theta$
$$= -\int_{\pi/2}^{0} \theta \cdot \sin 2\theta \, d\theta$$
$$= \int_0^{\pi/2} \theta \cdot \sin 2\theta \, d\theta \quad \left(\because \int_a^b f(x) dx = -\int_b^a f(x) dx\right)$$
$$= \left[-\theta \frac{\cos 2\theta}{2} + \frac{1}{4} \sin 2\theta\right]_0^{\pi/2}$$
$$= \left[-\frac{\pi}{2} \cdot \frac{1}{2}(-1) + 0\right] = \frac{\pi}{4}$$

(iii) At $\left(x=\frac{\pi}{2}\right)$,

$$f\left(\frac{\pi}{2}\right)=\int_0^1 \sin^{-1}\sqrt{t}\,dt+0$$

Let $\quad t=\sin^2\theta,\ dt=\sin 2\theta\, d\theta$

$$=\int_0^{\pi/2}\theta\cdot\sin 2\theta\cdot d\theta=\left[-\theta\cdot\frac{\cos 2\theta}{2}+\frac{\sin 2\theta}{4}\right]_0^{\pi/2}$$

$$=\left[-\frac{\pi}{2}\cdot\frac{1}{2}(-1)+0\right]=\frac{\pi}{4}$$

14. (a) Total sample events $n(S)=(2)^3=8$

Cases	Value	Condition for imaginary roots $b^2-4ac<0$
H, T, T	1, 2, 2	$(2)^2-4(1)(2)<0$
H, H, T	1, 1, 2	$(1)^2-4(1)(2)<0$
H, T, H	1, 2, 1	$(2)^2-4(1)(1)=0$
H, H, H	1, 1, 1	$(1)^2-4(1)(1)<0$
T, H, H	2, 1, 1	$(1)^2-4(2)(1)<0$
T, T, H	2, 2, 1	$(2)^2-4(2)(1)<0$
T, H, T	2, 1, 2	$(1)^2-4(2)(2)<0$
T, T, T	2, 2, 2	$(2)^2-4(2)(2)<0$

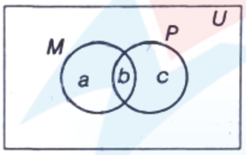
$\therefore$ Total favourable events $n(E)=7$

$\therefore$ Required probability $=\frac{n(E)}{n(S)}=\frac{7}{8}$

15. (d) Given $U=100$

$$a+b=55 \quad \ldots(i)$$

$$b+c=67 \quad \ldots(ii)$$



and $\quad a+b+c=100 \quad \ldots(iii)$

From Eqs. (i) and (iii),

$$(a+b)+c=100$$

$\Rightarrow \quad 55+c=100$

$\Rightarrow \quad c=100-55=45$

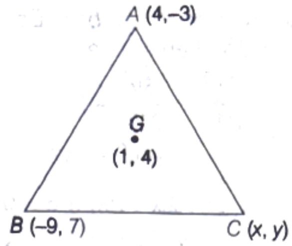
Hence, the number of students passed in Physics only is 45.

16. (c) We know that,

Centroid of the triangle,

$$G=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)=(1,4)$$

$$\Rightarrow \left\{\frac{4-9+x}{3},\frac{-3+7+y}{3}\right\}=(1,4)$$



$$\Rightarrow \left(\frac{x-5}{3},\frac{y+4}{3}\right)=(1,4)$$

$\Rightarrow \quad x-5=3 \Rightarrow x=8$

and $\quad y+4=12 \Rightarrow y=8$

So, third vertex of a ΔABC is (8, 8).

Now, area of $\Delta ABC=\frac{1}{2}\begin{vmatrix}4 & -3 & 1\\ -9 & 7 & 1\\ 8 & 8 & 1\end{vmatrix}$

Use $R_2\to R_2-R_1,\quad R_3\to R_3-R_1$,

$$=\frac{1}{2}\begin{vmatrix}4 & -3 & 1\\ -13 & 10 & 0\\ 4 & 11 & 0\end{vmatrix}$$

Expand with respect C_3

$$=\frac{1}{3}|\{-143-40\}|=\frac{1}{2}|-183|=\frac{183}{2}$$

17. (b) The equation of an ellipse whose major axis along x-axis is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \ldots(i)$$

Eq. (i) passes through the points (4, 3) and (– 1, 4), then

$$\frac{16}{a^2}+\frac{9}{b^2}=1 \quad \ldots(ii)$$

and $\quad \frac{1}{a^2}+\frac{16}{b^2}=1 \quad \ldots(iii)$

From Eqs. (ii) and (iii),

$$16\left(1-\frac{16}{b^2}\right)+\frac{9}{b^2}=1$$

$$\Rightarrow \frac{9}{b^2}-\frac{256}{b^2}=1-16$$

$$\Rightarrow \frac{247}{b^2}=15$$

$$\therefore b^2=\frac{247}{15}$$

From Eq. (iii),

$$\frac{1}{a^2}=1-\frac{16}{b^2}=1-\frac{15}{247}\times 16$$

$$\Rightarrow \frac{1}{a^2}=\frac{247-240}{247}=\frac{7}{247}$$

$$\Rightarrow \left(a^2=\frac{247}{7}\right)$$

Now, put the value of a^2 and b^2 in Eq. (i) and get the required equation of an ellipse

$$\frac{7x^2}{247} + \frac{15y^2}{247} = 1$$

$\Rightarrow \quad 7x^2 + 15y^2 = 247$

18. (a) Let $S_1 \equiv x^2 + y^2 + 2x + 2ky + 6 = 0$

Here $g_1 = 1$, $f_1 = k$, $C_1 = 6$, Centre $\rightarrow (-1, -k)$

and $S_2 \equiv x^2 + y^2 + 2ky + k = 0$

Here, $g_2 = 0$, $f_2 = k$ and $C_2 = k$, Centre $\rightarrow (0, -k)$

If two circles intersect orthogonally, then

(Distance between two centres)^2

= (Radius of circle S_1)^2 + (Radius of circle S_2)^2

$(-1-0)^2 + (-k+k)^2 = (\sqrt{1+k^2-6})^2 + (\sqrt{0+k^2-k})^2$

$\Rightarrow \quad 1 + 0 = (k^2 - 5) + (k^2 - k)$

$\Rightarrow \quad 2k^2 - k - 6 = 0$

$\Rightarrow \quad 2k^2 - 4k + 3k - 6 = 0$

$\Rightarrow \quad 2k(k-2) + 3(k-2) = 0$

$\Rightarrow \quad (k-2)(2k+3) = 0$

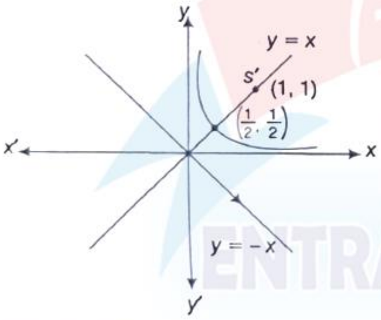
$\therefore \quad k = -\frac{3}{2}$ or 2

19. (a) $x^2 + y^2 - 2xy - 4(x + y - 1) = 0$

$\Rightarrow \quad (x-y)^2 = 4\{(x+y) - 1\}$

Here, $\quad x - y = 0 \quad$ ...(i)

and $\quad x + y = 1 \quad$ ...(ii)



On solving, we get

$$x = y = \frac{1}{2}$$

$\therefore$ Centre of parabola $= \left(\frac{1}{2}, \frac{1}{2}\right)$

Then, its focus, $S' = \left(2 \times \frac{1}{2}, 2 \times \frac{1}{2}\right)$

$= (1, 1)$

20. (d) Given, **a**, **b** and **c** are unit vectors.

$\therefore \quad |\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$

Now, we have

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$$

$\therefore \quad |\mathbf{a} + \mathbf{b} + \mathbf{c}|^2 = 0$

$\Rightarrow |\mathbf{a}|^2 + |\mathbf{b}|^2 + |\mathbf{c}|^2 + 2(\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a}) = 0$

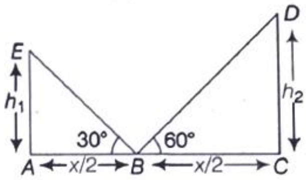
$\Rightarrow 1 + 1 + 1 + 2(\mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a}) = 0$

$\Rightarrow \quad \mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{c} + \mathbf{c}\cdot\mathbf{a} = -\frac{3}{2}$

21. (b) In ΔABE,

$$\tan 30° = \frac{h_1}{x/2} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad x = 2\sqrt{3}\, h_1 \quad$ ...(i)



and in ΔBCD,

$$\tan 60° = \frac{h_2}{x/2} = \sqrt{3}$$

$\Rightarrow \quad x = \frac{2h_2}{\sqrt{3}} \quad$ ...(ii)

From Eqs. (i) and (ii),

$$2\sqrt{3}\, h_1 = \frac{2h_2}{\sqrt{3}}$$

$\Rightarrow \quad \frac{h_1}{h_2} = \frac{1}{3} \Rightarrow h_1 : h_2 = 1 : 3$

22. (a) Given that, the vectors $\mathbf{a} = (1, x, -2)$ and $\mathbf{b} = (x, 3, -4)$ are mutually perpendicular.

$\therefore \quad (1)x + 3(x) + (-4)(-2) = 0$

$\Rightarrow \quad x + 3x + 8 = 0$

$\Rightarrow \quad 4x = -8$

$\therefore \quad x = -2$

23. (c) Given function, $f(x) = \begin{cases} \sin x, & \text{if } x \le \frac{\pi}{2} \\ ax, & \text{if } x > \frac{\pi}{2} \end{cases}$

and the function is continuous at $\frac{\pi}{2}$.

$\therefore \quad \lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \quad \lim_{x \to \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$

$\Rightarrow \quad \lim_{h \to 0} a\left(h + \frac{\pi}{2}\right) = \sin\frac{\pi}{2}$

$\Rightarrow \quad a\left(0 + \frac{\pi}{2}\right) = 1$

$\therefore \quad a = \frac{2}{\pi}$

24. (b) By given condition, we get

Let $f(x) = x + \frac{1}{x}$ ...(i)

On differentiating w.r.t. x, we get

$$f'(x) = 1 - \frac{1}{x^2}$$

For max or min of $f(x)$,

Put $f'(x) = 0$

$\Rightarrow 1 - \frac{1}{x^2} = 0$

$\Rightarrow \frac{(x^2-1)}{x^2} = 0$ $(\because x \neq 0)$

$\Rightarrow (x-1)(x+1) = 0$

$\Rightarrow x = 1$ or -1

Now, $f''(x) = \frac{2}{x^3}$

at $x = -1$, $f''(-1) = -2$ (max)

at $x = 1$, $f''(1) = 2$ (min)

So, $f(x)$ is min at $(x = 1)$ and its minimum value at $(x = 1)$ is

$$f(1) = 1 = \frac{1}{1} = 2$$

or Let $f(x) = x + \frac{1}{x}$

$\because$ AM $\geq$ GM

$\Rightarrow \frac{x + \frac{1}{x}}{2} \geq \left(x \cdot \frac{1}{x}\right)^{1/2} \Rightarrow \left(x + \frac{1}{2}\right) \geq 2$

Min of $f(x)$ is 2.

25. (a) Given, $\cos(\alpha+\beta) = \frac{4}{5}$

and $\sin(\alpha-\beta) = \frac{5}{13}$ where, $0 < \alpha, \beta < \frac{\pi}{4}$

Using the identity $\sin^2\theta + \cos^2\theta = 1$

Now, $\sin(\alpha+\beta) = \sqrt{1-\cos^2(\alpha+\beta)} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}}$

$\therefore \sin(\alpha+\beta) = \frac{3}{5}$

and $\cos(\alpha-\beta) = \sqrt{1-\sin^2(\alpha-\beta)}$

$$= \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}}$$

$\therefore \cos(\alpha-\beta) = \frac{12}{13}$

Now, $\tan 2\alpha = \tan\{(\alpha+\beta)+(\alpha-\beta)\}$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)}$$

$$= \frac{\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} + \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}}{1-\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}\cdot\frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)}}$$

$$= \frac{\frac{3}{5}\times\frac{5}{4}+\frac{5}{13}\times\frac{13}{12}}{1-\left(\frac{3}{5}\times\frac{5}{4}\right)\left(\frac{5}{13}\times\frac{13}{12}\right)} = \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{15}{4\cdot 12}}$$

$$= \frac{(9+5)}{12\left(1-\frac{15}{4\times 12}\right)} = \frac{14}{12-\frac{15}{4}} = \frac{14\times 4}{33} = \frac{56}{33}$$

26. (b) $\therefore$ Required number of ways $= \frac{9!}{2!\,2!}$

$$= \frac{362880}{2\cdot 2} = 90720$$

27. (a) Given, $A - B = \frac{\pi}{4}$

$\Rightarrow \tan(A-B) = \tan\frac{\pi}{4} = 1$

$\Rightarrow \frac{\tan A - \tan B}{1+\tan A\cdot\tan B} = 1$

$\Rightarrow \tan A - \tan B = 1 + \tan A\cdot\tan B$

$\Rightarrow 1 - \tan A + \tan B + \tan A\cdot\tan B = 0$

$\Rightarrow 2 = 1 + \tan A - \tan B - \tan A\tan B$

$\Rightarrow 2 = (1-\tan B) + \tan A(1-\tan B)$

$\Rightarrow 2 = (1-\tan B)(1+\tan A)$

28. (d) Given, $P(E)$ = Probability of event E

and $P(A) = 1$, $P(B) = \frac{1}{2}$

Now, $P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)\,P(B)}{P(B)} = P(A) = 1$

and $P\left(\frac{B}{A}\right) = \frac{P(A\cap B)}{P(A)} = \frac{P(A)\,P(B)}{P(A)} = P(B) = \frac{1}{2}$

29. (a) The number of arrangements of 3 English letters with repetitions allowed

$$= 26\cdot 26\cdot 26 = (26)^3$$

The number of arrangements of 4 digits with repetition allowed

$$= 10\cdot 10\cdot 10\cdot 10 = (10)^4$$

$\therefore$ Required number of different licence plates

$$= (26)^3 \times (10)^4$$

30. (b) $\because$ $1° < 1 \Rightarrow \sin 1° < \sin 1$

31. (c) Let $\mathbf{A} = \mathbf{a} + 2\mathbf{b} + 3\mathbf{c}$

$\mathbf{B} = \lambda\mathbf{b} + 4\mathbf{c}$

$\mathbf{C} = (2\lambda - 1)\mathbf{c}$

Since, $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are non-coplanar vectors.

$\therefore \quad [\mathbf{A\,B\,C}] \neq 0$

$\Rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & (2\lambda - 1) \end{vmatrix} \neq 0$

$\Rightarrow \quad 1 \cdot \lambda \cdot (2\lambda - 1) \neq 0$

$\Rightarrow \quad \lambda \neq 0, \frac{1}{2}$

Hence, all except two values of λ.

32. (a) Since, standard deviation (SD) < Range

$\Rightarrow \quad \sigma \leq (b-a)$

$\Rightarrow \quad \sigma^2 \leq (b-a)^2$

$\Rightarrow \quad (b-a)^2 \geq \sigma^2$

or $\quad (b-a)^2 \geq \text{Var}(X)$

33. (b) Given system of homogeneous linear equation are

$$x + \omega^2 y + \omega z = 0$$
$$\omega x + y + \omega^2 z = 0$$
$$\omega^2 x + \omega y + z = 0$$

Let coefficient matrix

$$A = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega & 1 & \omega^2 \\ \omega^2 & \omega & 1 \end{bmatrix} \quad \left\{\begin{matrix} \because \omega^3 = 1 \\ 1 + \omega + \omega^2 = 0 \end{matrix}\right\}$$

Use operation,

$$R_2 \to R_2 - \omega R_1, \quad R_3 \to R_3 - \omega^2 R_1$$

$$A \sim \begin{bmatrix} 1 & \omega^2 & \omega \\ 0 & 1-\omega^3 & \omega^2 - \omega^2 \\ 0 & \omega - \omega^4 & 1 - \omega^3 \end{bmatrix} \sim \begin{bmatrix} 1 & \omega^2 & \omega \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So, $\quad f(A) = r = 1$

and number of unknowns, $n = 3$

Since, $r < n$, so the system of equations is consistent and has more than one solution.

34. (c) Given that, $x = \log_a bc = \frac{\log bc}{\log a}$

$$y = \log_b ca = \frac{\log ca}{\log b}$$

and $\quad z = \log_c ab = \frac{\log ab}{\log c}$

$$\therefore \quad \frac{1}{1+x} + \frac{1}{1+y} + \frac{1}{1+z} = \frac{1}{1 + \frac{\log bc}{\log a}} + \frac{1}{1 + \frac{\log ca}{\log b}} + \frac{1}{1 + \frac{\log ab}{\log c}}$$

$$= \frac{\log a}{\log abc} + \frac{\log b}{\log abc} + \frac{\log c}{\log abc} = \frac{\log abc}{\log abc} = 1$$

35. (c) Given, $2^a = 3^b = 6^{-c} = K$ (say)

$\Rightarrow \quad a = \log_2 K, \quad b = \log_3 K, \quad c = -\log_6 K$

$\Rightarrow \quad a = \frac{\log K}{\log 2}, b = \frac{\log K}{\log 3}, \quad c = -\frac{\log K}{\log 6}$

$\Rightarrow \quad \log 2 + \log 3 = -\frac{\log K}{c} \quad (\because \log 6 = \log 2 + \log 3)$

$\Rightarrow \quad \frac{\log K}{a} + \frac{\log K}{b} = -\frac{\log K}{c}$

$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0 \quad (\because \log K \neq 0)$

$\Rightarrow \quad \frac{bc + ca + ab}{abc} = 0 \quad (\because abc \neq 0)$

$\Rightarrow \quad ab + bc + ca = 0$

36. (b) We know that, the eccentricity of hyperbola is

$$b^2 = a^2(e^2 - 1)$$

$\Rightarrow \quad \frac{b^2}{a^2} = e^2 - 1$

$\Rightarrow \quad e^2 = \frac{a^2 + b^2}{a^2}$

$\Rightarrow \quad \frac{1}{e^2} = \frac{a^2}{a^2 + b^2} \quad \ldots\text{(i)}$

and the eccentricity of its conjugate

$$a^2 = b^2(e'^2 - 1)$$

$\Rightarrow \quad \frac{a^2}{b^2} = e'^2 - 1$

$\Rightarrow \quad e'^2 = \frac{a^2 + b^2}{b^2}$

$\Rightarrow \quad \frac{1}{e'^2} = \frac{b^2}{a^2 + b^2} \quad \ldots\text{(ii)}$

On adding Eqs. (i) and (ii), we get

$$\frac{1}{e^2} + \frac{1}{e'^2} = \frac{a^2}{a^2 + b^2} + \frac{b^2}{a^2 + b^2} = \frac{a^2 + b^2}{a^2 + b^2}$$

$\Rightarrow \quad \frac{1}{e^2} + \frac{1}{e'^2} = 1$

37. (a) Here, $p = \frac{1}{2}$ and $q = \frac{1}{2}$

Now, by binomial distribution,

$$= {}^nC_1(p)^1(q)^{n-1} + {}^nC_3(p)^3(q)^{n-3} + {}^nC_5(p)^5(q)^{n-1} + \ldots$$

$$= {}^nC_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1} + {}^nC_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^{n-3} + {}^nC_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{n-5} + \ldots$$

$$= {}^nC_1\left(\frac{1}{2}\right)^n + {}^nC_3\left(\frac{1}{2}\right)^n + {}^nC_5\left(\frac{1}{2}\right)^n + \ldots$$

$= \left(\frac{1}{2}\right)^n \{{}^nC_1 + {}^nC_3 + {}^nC_5 + \dots\}$

$= \frac{1}{2^n} \cdot (2^{n-1}) = \frac{1}{2}$

38. (a) Given, $\sin(\pi \cos\theta) = \cos(\pi \sin\theta)$

$\Rightarrow \quad \cos(\pi \sin\theta) = \cos\left\{\frac{\pi}{2} - (\pi\cos\theta)\right\}$

$\Rightarrow \quad \pi\sin\theta = \pm\left[\frac{\pi}{2} - \pi\cos\theta\right]$

$\Rightarrow \quad \sin\theta + \cos\theta = \frac{1}{2}$ (taking +ve sign)

$\Rightarrow \quad (\sin\theta + \cos\theta)^2 = \left(\frac{1}{2}\right)^2$

$\Rightarrow \quad (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cdot\cos\theta = \frac{1}{4}$

$\Rightarrow \quad 1 + \sin 2\theta = \frac{1}{4}$

$\Rightarrow \quad \sin 2\theta = -\frac{3}{4}$ ...(i)

$\Rightarrow \quad \sin\theta = -\frac{1}{2} + \cos\theta$ (Taking –ve sign)

$\Rightarrow \quad \cos\theta - \sin\theta = \frac{1}{2}$

On squaring both sides,

$(\cos\theta - \sin\theta)^2 = \left(\frac{1}{2}\right)^2$

$\Rightarrow \quad \cos^2\theta + \sin^2\theta - 2\sin\theta\cdot\cos\theta = \frac{1}{4}$

$\Rightarrow \quad 1 - \sin 2\theta = \frac{1}{4}$

$\Rightarrow \quad \sin 2\theta = \frac{3}{4}$ ...(ii)

$\therefore$ From Eqs. (i) and (ii), we get

$\sin 2\theta = \pm\frac{3}{4}$

39. (a) For pentagon,

Number of sides, $n = 5$

Number of diagonals $= {}^5C_2 - 5 = \frac{5\cdot 4}{2} - 5$

$= 10 - 5 = 5$

Hence, number of sides is equal to number of diagonal of pentagon.

40. (d) By condition, using binomial distribution,

${}^{100}C_{50} P^{50}(1-P)^{50} = {}^{100}C_{51} P^{51}(1-P)^{49}$

$\Rightarrow \quad \frac{100!}{50!\,50!}(1-P) = \frac{100!}{51!\,49!}\cdot P$

$\Rightarrow \quad \frac{1}{50}(1-P) = \frac{P}{51}$

$\Rightarrow \quad 51 - 51P = 50P$

$\Rightarrow \quad 101P = 51$

$\therefore \quad P = \frac{51}{101}$

41. (d) Given equation is

$(\cos P - 1)x^2 + \cos P\cdot x + \sin P = 0$

Since, the equation has real roots.

So, $\quad \Delta = B^2 - 4AC \geq 0$

$\Rightarrow \quad \cos^2 P - 4(\cos P - 1)\sin P \geq 0$

$\Rightarrow \quad \cos^2 P - 4\sin P\cdot\cos P + 4\sin P \geq 0$

$\Rightarrow$ For real value of P

$(-4\sin P)^2 - 4\cdot 1\cdot(4\sin P) > 0$

$\Rightarrow \quad 16\sin^2 P - 16\sin P > 0$

$\Rightarrow \quad \sin P(\sin P - 1) > 0$

$\Rightarrow \quad \sin P > \sin 0 \quad$ or $\quad \sin P > \sin\frac{\pi}{2}$

$\Rightarrow \quad P > n\pi + (-1)^n\cdot 0 \quad$ or $\quad P > n\pi + (-1)^n\frac{\pi}{2}$

$\Rightarrow \quad P \in (0, \pi) \quad$ or $\quad$ (no possible)

42. (c) Let $f(x) \equiv 3x^5 + 15x - 8 = 0$

For positive roots,

$f(x) = + \quad \underbrace{+ \quad -}_{\text{1 change}} \quad = 1$

For negative roots,

$f(-x) = \underbrace{-3x^5 - 15x - 8}_{\text{no change}} = 0$

$\therefore$ Real roots = Number of positive roots – Number of negative roots $= 1 - 0 = 1$

43. (d) The given system of homogeneous equation

$3x + Ky - 2z = 0$

$x + Ky + 3z = 0$

$2x + 3y - 4z = 0$

For non-trivial solution,

$$\begin{vmatrix} 3 & K & -2 \\ 1 & K & 3 \\ 2 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow 3(-4K - 9) - K(-4 - 6) + 2(-3 + 2K) = 0$

$\Rightarrow -12K - 27 + 10K - (+6) + 4K = 0$

$\Rightarrow \quad +2K - 33 = 0$

$\therefore \quad K = +\frac{33}{2}$

44. (a) Given, $x = \log_3 5$, $y = \log_{17} 25$

$$x = \frac{\log 10 - \log 2}{\log 3}, \quad y = \frac{2\log 10 - 2\log 2}{\log 17}$$

$$x = \frac{0.6990}{0.4771}, \quad y = \frac{1.3980}{1.2296}$$

$\Rightarrow$ $x = 1.465$, $y = 1.136$ $\quad (\therefore x > y)$

45. (b) Given, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

Now, $A^2 = A \cdot A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

....

....

$A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

46. (d) $\therefore$ Required probability

$$= 1 - \left(1 - \frac{1}{2}\right) \cdot \left(1 - \frac{1}{3}\right) \cdot \left(1 - \frac{1}{4}\right)$$

$$= 1 - \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

47. (c) Let $y = x^x$

Taking log on both sides, we get

$$\log y = x \log x$$

On differentiating,

$$\frac{1}{y}\frac{dy}{dx} = x \cdot \frac{1}{x} + \log x$$

$$\frac{dy}{dx} = y(1 + \log x) = x^x \cdot (1 + \log x) \quad \ldots(i)$$

For decreasing of y,

Here, $\frac{dy}{dx} < 0$

$x^x \cdot (1 + \log x) < 0$ (but $x^x \not< 0$ and $x > 0$)

$\Rightarrow$ $1 + \log x < 0$

$\Rightarrow$ $\log x < -1$

$\Rightarrow$ $\log x < \log e^{-1}$

$\Rightarrow$ $x < \frac{1}{e}$ and $x > 0$

$\therefore$ $x \in \left(0, \frac{1}{e}\right)$

48. (b) Given expression

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$$

$\Rightarrow$ $\mathbf{a} + \mathbf{b} = -\mathbf{c}$

On squaring both sides,

$\Rightarrow$ $(\mathbf{a} + \mathbf{b})^2 = (-\mathbf{c})^2$

$\Rightarrow$ $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b}) = (-\mathbf{c}) \cdot (-\mathbf{c})$

$\Rightarrow$ $(\mathbf{a} \cdot \mathbf{a}) + (\mathbf{b} \cdot \mathbf{a}) + (\mathbf{a} \cdot \mathbf{b}) + (\mathbf{b} \cdot \mathbf{b}) = (\mathbf{c} \cdot \mathbf{c})$

$\Rightarrow$ $\mathbf{a}^2 + 2\mathbf{a} \cdot \mathbf{b} + \mathbf{b}^2 = \mathbf{c}^2$ $\quad \because (\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a})$

$\Rightarrow$ $|\mathbf{a}|^2 + 2\mathbf{a} \cdot \mathbf{b} + |\mathbf{b}|^2 = |\mathbf{c}|^2$ $\quad \because (\mathbf{a}^2 = |\mathbf{a}|^2)$

$\Rightarrow$ $(3)^2 + 2\mathbf{a} \cdot \mathbf{b} + (5)^2 = (7)^2$

$\because |\mathbf{a}| = 3, |\mathbf{b}| = 5$ and $|\mathbf{c}| = 7$

$\Rightarrow$ $2\mathbf{a} \cdot \mathbf{b} = 49 - 25 - 9$

$\Rightarrow$ $2\mathbf{a} \cdot \mathbf{b} = 15$

$\Rightarrow$ $2 \cdot |\mathbf{a}||\mathbf{b}| \cos\theta = 15$

Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$.

$\Rightarrow$ $2 \cdot 3 \cdot 5 \cos\theta = 15$

$\Rightarrow$ $\cos\theta = \frac{1}{2} = \cos 60$

$\therefore$ $\theta = \frac{\pi}{3}$

49. (c) $\because \theta \in [0, \pi]$

Now, $\frac{|\mathbf{a} \times \mathbf{b}|}{\mathbf{a} \cdot \mathbf{b}} = \frac{||\mathbf{a}||\mathbf{b}|\sin\theta\,(\mathbf{n})|}{|\mathbf{a}||\mathbf{b}|(-\cos\theta)}$

$$= \frac{|\mathbf{a}||\mathbf{b}||\sin\theta||\mathbf{n}|}{|\mathbf{a}||\mathbf{b}|(-\cos\theta)}$$

$$= \frac{\sin\theta \cdot 1}{-\cos\theta} = -\tan\theta$$

($\cos\theta$ in second quadrant is negative)

50. (c) Given that,

$f(a + b) = f(a) \times f(b)$ $\quad \ldots(i)$

and $f(5) = 2$, $f'(0) = 3$

By definition,

$$f'(5) = \lim_{h \to 0} \frac{f(5 + h) - f(5)}{h}$$

$$f'(5) = \lim_{h \to 0} \frac{f(5) \times f(h) - f(5)}{h}$$

$$f'(5) = f(5) \cdot \lim_{h \to 0} \frac{f(h) - 1}{h}$$

By 'L'hospital rule,

$f'(5) = f(5) \cdot f'(0)$

$\Rightarrow$ $f'(5) = 2 \times 3 = 6$

51. (a) Let the distance covered by him is x km, then by condition,

$$\frac{x}{4} - \frac{x}{5} = \frac{12}{60}$$

$\Rightarrow$ $\frac{x}{20} = \frac{1}{5}$

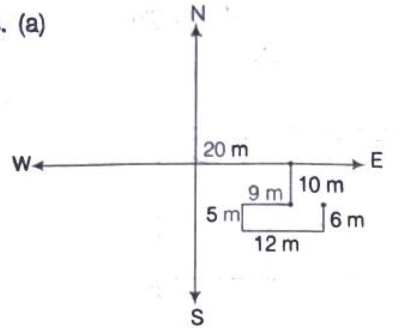
$\therefore$ $x = 4$ km

52. (b) Given series, 3, 6, 6, 12, 9, ..., 12

Split the given series into two parts

3 $\underset{+3}{\rule{0pt}{0pt}}$ 6 $\underset{+3}{}$ 9 $\underset{+3}{}$ 12 $\qquad$ 6 $\underset{+6}{}$ 12 $\underset{+6}{}$ [18]

53. (a)



Hence, North direction is the man facing.

54. (b) Let x and y be the certain number of males and females.

Then, by condition,

$$x = \frac{1}{2}(y - 15)$$

$\Rightarrow \quad 2x = y - 15$

$\Rightarrow \quad 2x - y = -15 \quad \ldots(i)$

and $\quad 5(x - 45)) = y$

$\Rightarrow \quad 5x - y = 225 \quad \ldots(ii)$

On subtracting Eq. (i) from Eq. (ii), we get

$3x = 240 \Rightarrow x = 80$

$\therefore$ Number of males = 80

55. (b) 6, 12, 21, [33], 48 (+6, +9, +12, +15)

Solution (Q.Nos. 56-58)

By condition D E C A F B (Shortest)
↓
(Longest)

56. (c) Between D and C

57. (d) C

58. (d) F

59. (a) x, y, z are distinct integers.

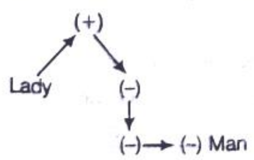
and x and y are odd positive integers and z is even positive integers.

Then, $\quad (x - z) = $ Odd number

$(x - z)^2 = $ Odd positive number

and $(x - z)^2 \cdot y = \text{Odd} \times \text{Even} = $ Odd number

60. (d)



So, man is nephew of the lady.

61. (d) 2, 9, 28, 65, 126, 216, 344 .... (2^3+1, 3^3+1, 4^3+1, 5^3+1, 6^3+0, 7^3+1)

or $2 = 1^3 + 1$, $9 = 2^3 + 1$, $28 = 3^3 + 1$, $65 = 4^3 + 1$,

$126 = 5^3 + 1$ and $344 = 7^3 + 1$

But $216 = 6^3 + 0$ which is odd number among them.

62. (d) Let the total number of students before joining new students = x.

$\Rightarrow$ After joining new 120 students = $x + 120$

Now, by condition,

$x \times 40 + 120 \times 32 = (x + 120) \times 36$

$\Rightarrow \quad 40x + 3840 = 36x + 4320$

$\Rightarrow \quad 4x = 480$

$\therefore \quad x = 120$

$\therefore$ Total number of students = $x + 120 = 120 + 120 = 240$

63. (c) By given condition, we get the required order (sequence) of letters from the lowest value to the highest value is

T< U< S< Q< R< P< V

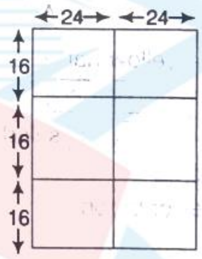
i.e., TUSQRPV

64. (a) From option (a),

Let the number of tiles = 6

$\therefore$ Total length = 48

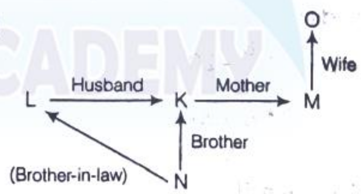
and total breadth = 48



Since, length = breadth

$\therefore$ Number of tiles form a square = 6

65. (d)



66. (a) Required total number of games played is 12.

Solutions (Q.Nos. 67-69)

(i) A causes B or C but not both.
(ii) F occurs only if B occurs.
(iii) D occurs if B or C occurs.
(iv) E occurs only if C occurs.
(v) C occurs only if E or F occurs
(vi) D causes G or H or both
(vii) H occurs if E occurs.
(viii) G occurs if F occurs.

67. (c) From Statement (i), A causes B or C but not both. From Statement (ii), F occurs only if B occurs and from Statement (iii), D occurs if B or C occur. It means I and II may occur. From Statements (vi) and (vii), II and III are may occur. So, we conclude that I and III or II and III may occur but not both occur.

68. (b) From Statement (ii) that F occurs only if B occurs and from Statement (viii) that if G occurs if F occurs it means if B occurs G must occur.

69. (b) From Statement (v), that J occurs only if E or F occurs. From Statement (ii), F occurs only if B occurs and from Statement (iv), E occurs only if C occurs it means if J occurs either B or C must have occurs.

70. (c) $R \xrightarrow{-2} P \quad S \xrightarrow{-2} Q$

$O \xrightarrow{+2} P \quad L \xrightarrow{+2} N$

$\Rightarrow$

$A \xrightarrow{-2} Y \quad O \xrightarrow{-2} M$

$S \xrightarrow{+2} U \quad P \xrightarrow{+2} R$

$P \xrightarrow{-2} N$

$T \xrightarrow{-2} R \quad Y \xrightarrow{+2} A$

71. (b) lelibroon $\longrightarrow$ yellow hat

pleka $\longrightarrow$ flower garden

froti mix $\longrightarrow$ garden salad

$\therefore$ Pleka $\longrightarrow$ flower

yellow $\longrightarrow$ leli or broon

By option,

yellow flower $\longrightarrow$ lelipleka

72. (c) $E = 6 - 9 + 8 * \frac{3}{20}$

By given condition,

$$E = 6 + 9 * \frac{8}{3} - 20$$
$$E = 6 + 3 * 8 - 20$$
$$E = 6 + 24 - 20$$
$$E = 6 + 4 = 10$$

73. (b) Let in a month of January.

(4 times) Friday $\longrightarrow$ 25, 18, 11, 4 (dates)

(4 times) Monday $\longrightarrow$ 28, 21, 14, 7 (dates)

Then, required dates of Sunday,

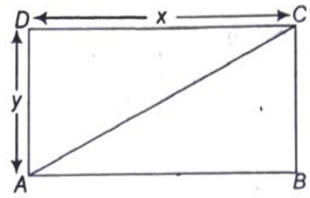
Sunday $\longrightarrow$ 27, 20, 13, 6

So, Sunday of the week did the 20th of January fall that year.

74. (b) Krishna is "father-in-law" of that girl.

75. (d) Let longer side $= x = DC$

and shorter side $= y = AD$



Now, by condition,

$$AC = y + \frac{x}{2}$$

Now, In ΔACD,

$$AC^2 = AD^2 + CD^2 \text{ (by Pythagoras theorem)}$$

$$\Rightarrow \left(y + \frac{x}{2}\right)^2 = y^2 + x^2$$

$$\Rightarrow y^2 + \frac{x^2}{4} + xy = x^2 + y^2$$

$$\Rightarrow \frac{x^2}{4} + xy - x^2 = 0$$

$$\Rightarrow x\left\{\frac{x}{4} + y - x\right\} = 0$$

$$\Rightarrow x\left(y - \frac{3x}{4}\right) = 0$$

$$\Rightarrow \frac{y}{x} = \frac{3}{4} \quad \because x \neq 0$$

76. (c) SNIP (NICE) PACE

TEAR (EAST) FAST

TRAY (RARE) FIRE

POUT (OURS) CARS

$\therefore$ CANE (AN+TS) BATS

ANTS

77. (c) From the statements, we clearly say that the reason behind the nearsightedness of the children is caused by the visual stress required by reading and other class work.

Solutions (Q.Nos. 78-80)

Randy	Vaccuming	Monday
Sally	Dusting	Tuesday
Terry	Sweeping	Wednesday
Uma	Mopping	Thursday
Vernon	Laundry	Friday

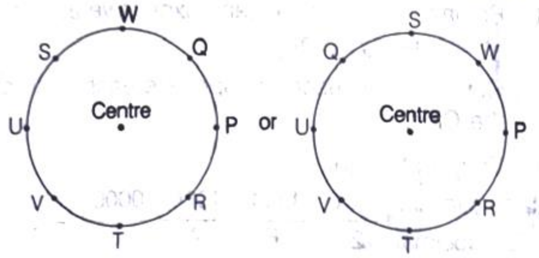
78. (d) Sweeping

79. (b) Monday

80. (c) Tuesday

Solutions (Q.Nos. 81-82)

According to the given data, we get the following figure



81. (a) R and V are not neighbours.

82. (d) The position of S is not fixed. So, data inadequate.

83. (c) Let the ten's place digit = x, then

By condition the unit place digit = $x + 3$

Now, according to question,

$$\frac{10x + (x + 3)}{x + x + 3} = \frac{4}{7} \Rightarrow \frac{1 + x + 3}{2x + 3} = \frac{4}{7}$$

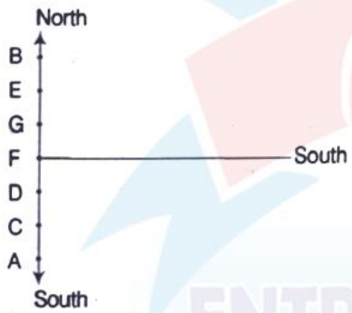
$$\Rightarrow \quad 1 + x + 3 = 8x + 12$$

$$\Rightarrow \quad 3x = 9 \Rightarrow x = 3$$

$\therefore$ Required number = $x(x + 3) = 3(3 + 3) = 36$

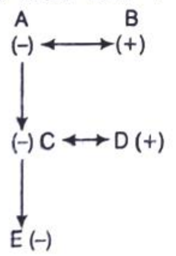
84. (c) After observation of given two dice, we get the number 5 is at the bottom of the dice, when number 1 is on the top.

85. (d) According to given data, we get the following figure



So, G is sitting third from North.

86. (a) Let '–' means 'male' and '+' means 'female'.



Two fathers (A, C)	Two mothers (B, D)
Two sons (C, E)	One father-in-law (A)
One mother-in-law (B)	One daughter-in-law (D)
One grandfather (A)	One grandmother (B)

One grandson (E)

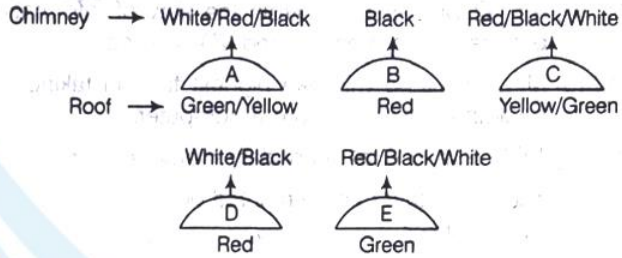
So, the minimum number of persons can be 5.

87. (a) According to the directions, the relation can be solved as

D ⟶ A ⟵ B ⟶ C
(–) (–)

So, A is brother of C.

Solutions (Q.Nos. 88-90)



88. (b) From the above diagram, it is clear that atleast two houses have red roofs.

89. (a) If house C has a yellow roof it means house D has a red roof. So, house D never has a red chimney. So, chimney of D will be of black colour, so colour of chimney of house E will be white.

90. (c) The maximum number of green roofs are 3.

91. (c) Ceiling is the correct word.

92. (d) Decieve is the wrongly spelt word, the correct spell is deceive.

93. (d) Controversial is most similar in meaning to the word 'Polemic'.

94. (c) Atrocities; development.

95. (b) The thief had escaped before the police came.

96. (c) Anne had to pay for everything because as usual, Peter left his wallet at home.

97. (d) Synonym of the word 'Meagre' is limited.

98. (d) Damaging the reputation.

99. (a) Antonym of word 'Timid' is bold.

100. (a) "If you would have" sentence has an error.

101. (b) Opposite in meaning to the word EXTRINSIC is Inherent.

102. (d) Idiom—To eat a humble pie.

Meaning–To say you are sorry for a mistake that you made.

103. (a) Word → Fabricate

Antonym → Construct

104. (a) The people with whom you socialise are called friends.

105. (a) Did you walk to school yesterday?

106. (d) There was no room in the railway compartment for additional passengers.

107. (b) And now for this evening's main headline; Britain wins another olympic gold medal.

108. (d) If she have known about his financial situation, she would helped him out.

109. (b) I am sure she can teach computers as well. She's not altogether new to the subject.

110. (a) You are trying to drag me in a controversy.

111. (b) An I/O processor controls the flow of information between main memory and I/O devices.

112. (d) Magnetic tape will take highest time in taking the backup of the data from a computer.

113. (d) ROM is a kind of secondary memory.

114. (a) The errors that can be pointed out by compilers are syntax errors.

115. (a)

116. (b) Required range is – 128 to + 127.

117. (c) Primary storage is fast and expensive as compared to secondary memory.

118. (a) Control unit is used to supervise each instruction in the CPU.

119. (c) From option (b),

Binary form	0010	1111	1010	0000	1100
Hexadecimal	2	F	A	O	C

$\therefore (2FAOC)_{16} = (00101111101000001100)_2$

From option (a),

$(195084)_{10} = (00101111101000001100)_2$

$= (2FAOC)_{16}$

120. (b) $(\underbrace{111}_{7}\ \underbrace{010}_{2})_2 \rightarrow$ Binary

$\rightarrow (72)_8 \rightarrow$ Octal.

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