ACME MCA ENTRANCE ACADEMY

NIMCET Previous year paper 2013

Included Subjects

Mathematics
Logical Reasoning
Computer
English

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NIMCET 2013

Mathematics:

1. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$ and $6A^{-1} = A^2 + cA + dI$, where A^{-1} is the inverse of A, I is the identity matrix, then (c, d) is

(A) $(-6, 11)$ (B) $(6, -11)$
(C) $(11, -6)$ (D) $(6, 11)$

2. If $a = \hat{j} - \hat{k}$ and $c = \hat{\imath} - \hat{j} - \hat{k}$, then vector b satisfying $(a \times b) + c = 0$ and $a.b = 3$ is

(A) $-\hat{\imath} + \hat{j} - 2\hat{k}$ (B) $2\hat{\imath} - \hat{j} + 2\hat{k}$
(C) $\hat{\imath} - \hat{j} - 2\hat{k}$ (D) $\hat{\imath} + \hat{\jmath} - 2\hat{k}$

3. Find the number of elements in the union of four sets A, B, C and D having 150, 180, 210 and 240 elements respectively, given that each pair of sets has 15 elements in common. Each triple of sets has 3 elements in common and $A \cap B \cap C \cap D = \Phi$.

(A) 616 (B) 512
(C) 111 (D) 702

4. If the straight line $ax + by + c = 0$ always passes through $(1, -2)$, then a, b and c are in

(A) AP (B) HP
(C) GP (D) None of these

5. A six faced die is biased one. It is thrice more likely to show an odd number than to show an even number. It is thrown twice. The probability that the sum of the numbers in the two throws is even, is

(A) $\frac{4}{8}$ (B) $\frac{5}{8}$
(C) $\frac{6}{8}$ (D) $\frac{7}{8}$

6. If $I_n = \int_0^{\pi/4} \tan^n\theta d\theta$ then $I_8 + I_6$ is equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{5}$
(C) $\frac{1}{6}$ (D) $\frac{1}{7}$

7. If ΔABC is a triangle whose area is $10\sqrt{3}$ units with side lengths $|AB| = 8$ units and $|AC| = 5$ units. Find the possible values of $\angle A$.

(A) 60° or 120° (B) 45° or 135°
(C) Only 30° (D) Only 90°

8. Person A can hit a target 4 times in 5 attempts. Person B can hit a target 3 times in 4 attempts. Person C can hit a target 2 times in 3 attempts. They fire a volley. The probability that the target is hit atleast two times, is

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$
(C) $\frac{5}{6}$ (D) 1

9. The value of the integral $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ is

(A) 0 (B) $-\frac{\pi}{4}$
(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

10. If ω is a cube root of unity, then find the value of $\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega^2+\omega & \omega & -\omega^2 \end{vmatrix}$.

(A) 3ω (B) -3ω
(C) $3\omega^2$ (D) $-3\omega^2$

11. If the vectors $2\hat{\imath} - 3\hat{\jmath}, \hat{\imath} + \hat{\jmath} - \hat{k}$ and $3\hat{\imath} - \hat{k}$ form three conterminus edges of a parallelopiped, then the volume of parallelopiped is

(A) 8 (B) 10
(C) 4 (D) 14

12. In a GP consisting of positive terms, each term equals the sum of the next two terms. Then, the common ratio of the GP is

(A) $\frac{(1-\sqrt{5})}{2}$ (B) $\frac{(\sqrt{5})}{2}$
(C) $\sqrt{5}$ (D) $\frac{(\sqrt{5}-1)}{2}$

13. If $f(x) = tan^{-1}\left[\frac{\sin x}{1+\cos x}\right]$, then what is the first derivative of $f(x)$?

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$
(C) 2 (D) -2

14. The solution of $\sin x + 1 = \cos x$, such that $0 \leq x \leq 2\pi$, is
(A) $0, \pi$ (B) $0, \frac{\pi}{2}$
(C) $\frac{\pi}{2}, \frac{3\pi}{2}$ (D) $0, \frac{3\pi}{2}$

15. If T_n denotes the number of triangles which can be formed by using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals
(A) 5 (B) 7
(C) 6 (D) 4

16. If $\bar{x}_1$ and $\bar{x}_2$ are the means of two distributions such that $\bar{x}_1 < \bar{x}_2$ and $\bar{x}$ is the mean of the combined distributions, then
(A) $\bar{x} < \bar{x}_1$ (B) $\bar{x} > \bar{x}_2$
(C) $\bar{x} = \frac{\bar{x}_1 < \bar{x}_2}{2}$ (D) $\bar{x}_1 < \bar{x} < \bar{x}_2$

17. The area enclosed within the curve $|x| + |y| = 1$ (in sq. units) is
(A) $\sqrt{2}$ (B) 1
(C) $\sqrt{3}$ (D) 2

18. Let $f(x)$ be a polynomial function of second degree and $f(1) = f(-1)$. If a, b and c are in AP, then $f'(a). f'(b)$ and $f'(c)$ are in
(A) GP (B) HP
(C) AGP (D) AP

19. Find the point at which the tangent to the curve $y = \sqrt{4x - 3} - 1$ has its slope $\frac{2}{3}$.
(A) $(3, 3)$ (B) $(3, 2)$
(C) $(2, 3)$ (D) $(2, 2)$

20. Atul speaks truth in 70% and George speaks the truth in 60% cases. In what percentage of cases they are likely to contradict each other in stating the same fact?
(A) $\frac{13}{50}$ (B) $\frac{11}{50}$
(C) $\frac{23}{50}$ (D) $\frac{33}{50}$

21. A man observes the angle of elevation of the top of a mountain to be 30°. He walks 1000 ft nearer and finds the angle of elevation to be 45°. What is the distance of the first point of observation from the foot of the mountain?
(A) $500\sqrt{3}(\sqrt{3} + 1)$ ft (B) $500(\sqrt{3} + 1)$ ft
(C) $500(\sqrt{3} - 1)$ ft (D) $500\sqrt{3}(\sqrt{3} - 1)$ ft

22. The sum of n terms of an arithmetic series is 216. The value of the first term is n and the value of the nth term is $2n$. The common difference d is
(A) 1 (B) $\frac{2}{3}$
(C) $\frac{3}{2}$ (D) $\frac{12}{11}$

23. Force $3\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$ and $2\hat{\imath} + \hat{\jmath} - 3\hat{k}$ are acting on a particle and displace it from the point $2\hat{\imath} - \hat{\jmath} - 3\hat{k}$ to the point $4\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$, then the work done by the force is
(A) 18 units (B) 30 units
(C) 24 units (D) 36 units

24. The value of $9^{\frac{1}{3}}. 9^{\frac{1}{9}}. 9^{\frac{1}{27}} \ldots$ is
(A) 3 (B) 6
(C) 9 (D) None of these

25. The minimum value of the function $y = 2x^3 - 21x^2 + 36x - 20$ is
(A) -120 (B) -126
(C) -128 (D) None of these

26. In how many different ways can the letters of the word 'CORPORATION' be arranged, so that all the vowels always come together?
(A) 810 (B) 1440
(C) 2880 (D) 50400

27. If $\log_x y = 100$ and $\log_2 x = 10$, then the value of y is
(A) 2^{10} (B) 2^{100}
(C) 2^{1000} (D) 2^{10000}

28. The equation of the line parallel to the line $2x - 3y = 7$ and passing through the mid-point of the line segment joining the points $(1, 3)$ and $(1, -7)$ is
(A) $2x - 3y - 4 = 0$ (B) $2x - 3y + 4 = 0$
(C) $2x - 3y - 8 = 0$ (D) $2x - 3y + 8 = 0$

29. In a ΔABC, if $(c + a + b)(a + b - c) = ab$, then measure of $\angle C$ is
(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$
(C) $\frac{2\pi}{3}$ (D) None of these

30. The number of non-negative integers less than 1000 that contain the digit 1 are
(A) 9^2 (B) 9^3
(C) $10^2 - 9^2$ (D) $10^3 - 9^3$

31. The lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangent to the same circle. The radius of this

x	1	2	3
$P(X = x)$	0.3	0.4	0.3

circle is
(A) $\frac{3}{2}$ (B) $\frac{3}{4}$
(C) $\frac{4}{5}$ (D) $\frac{7}{10}$

32. The area of the parallelogram whose diagonals are $a = 3\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $b = \hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ is
(A) $10\sqrt{3}$ (B) $5\sqrt{3}$
(C) $10\sqrt{2}$ (D) $5\sqrt{2}$

33. If $\sin x + a\cos x = b$, then what is the expression for $|a\sin x - \cos x|$ in terms of a and b?
(A) $\sqrt{a^2 - b^2 - 1}$ (B) $\sqrt{a^2 + b^2 - 1}$
(C) $\sqrt{a^2 + b^2 + 1}$ (D) $\sqrt{a^2 - b^2 + 1}$

34. If A and B are two events such that $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(\bar{B}) = \frac{1}{2}$. then the events A and B are
(A) dependent (B) independent
(C) mutually exclusive (D) None of these

35. If the three vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}, \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $3\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$ are coplanar, then λ is
(A) -1 (B) -2
(C) -3 (D) -4

36. The equation of the base of an equilateral triangle is $x + y = 2$ and the vertex is $(2, -1)$. The length of the side of the triangle is
(A) $\sqrt{\frac{3}{2}}$ (B) $\sqrt{2}$
(C) $\sqrt{\frac{2}{3}}$ (D) $\sqrt{\frac{20}{3}}$

37. The total number of numbers that can be formed using the digits 3, 5 and 7 only, if no repetitions are allowed, is
(A) 39 (B) 105
(C) 15 (D) 27

38. If $x = a\cos t, y = b\sin t$, then $\frac{d^2y}{dx^2}$ is
(A) $-\frac{b^4}{a^2y^3}$ (B) $\frac{b^4}{a^2x^3}$
(C) $\frac{b}{ay^4}$ (D) $-\frac{a^4}{bx^3}$

39. A random variable x has the distribution law as given below:

The variance of the distribution is
(A) 0.4 (B) 0.6
(C) 2 (D) None of these

40. The value of $\tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta$ is
(A) $\cot\theta$ (B) $\tan\theta$
(C) $\sin\theta$ (D) $\cos\theta$

41. The sum of integers between 200 and 400, that are multiples of 7 is
(A) 8729 (B) 8700
(C) 8972 (D) 8279

42. $\lim_{x\to 0} \frac{\tan x - x}{x^2 \tan x}$ is equal to
(A) 0 (B) 1
(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

43. Two fair dice are tossed. What is the probability that the total score is a prime number?
(A) $\frac{1}{6}$ (B) $\frac{5}{12}$
(C) $\frac{1}{2}$ (D) $\frac{7}{9}$

44. Find the equation of the circle which passes through $(-1, 1)$ and $(2, 1)$ and having centre on the line $x + 2y + 3 = 0$.
(A) $2x^2 + 2y^2 - 2x + 7y - 13 = 0$
(B) $x^2 + y^2 - 2x + 7y - 13 = 0$
(C) $2x^2 + 2y^2 + 2x + 7y - 13 = 0$
(D) $x^2 + y^2 + 2x + 7y - 13 = 0$

45. If a, b and c are the position vectors of three vertices A, B and C of a triangle, respectively. Then, the area of this triangle is given by
(A) $\frac{1}{2}(a \times b).c$

(B) $\frac{1}{2}|a \times b + c \times c + c \times a|$
(C) $a \times b + b \times c + c \times a$
(D) None of these

46. The sum of the focal distances of any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentricity e, is given by
(A) $2ae$ (B) $2b$
(C) $2a$ (D) $2be$

47. If $\sin x + sin^2 x = 1$, then $\cos^2 x + cos^4 x$ is equal to
(A) 0 (B) 1
(C) -1 (D) 2

48. An experiment succeeds twice often as it falls. The probability that in the next six trials there will be at least four successes, is
(A) $\frac{240}{729}$ (B) $\frac{496}{729}$
(C) $\frac{220}{729}$ (D) $\frac{233}{729}$

49. Sum of 20 terms of the series $-1^2 + 2^2 - 3^2 + 4^2 - ..$ is
(A) 180 (B) 200
(C) 210 (D) 220

50. If $\tan \alpha = \frac{m}{m+1}$ and $\beta = \frac{1}{2m+1}$, then $a + b$ is equal to
(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{6}$ (D) π

Reasoning/Aptitude:

91. Out of the alternatives, choose the appropriate phrase to make the sentence meaningful. If you had joint accounts with __ who died, then you will be responsible for the bills.
(A) everybody (B) anyone
(C) everyone (D) someone

92. Choose the analogy that is closest in meaning to the pair: Diamond : Necklace
(A) Cars : Roads (B) Flowers : Bouquet
(C) Gold : Bangle (D) Books : Shop

93. Choose the suitable preposition for the blank to make a meaningful sentence.
Suresh is angry ….. his servant.
(A) about (B) on
(C) by (D) with

94. Choose the correct alternative for the sentence below:
The Earth is always revolving round the Sun.
(A) The Earth revolves round the Sun.
(B) The Earth is revolving round the Sun.
(C) The Earth revolving round the Sun.
(D) None of the above

95. Choose the word that best expresses the meaning of the given idiom: 'A close shave.'
(A) A clean shave (B) A narrow escape
(C) A guarded secret (D) A sudden fall

96. Pick the part of the sentence that has an error.
My elder brother is a MA whereas I am only a BA
(A) My elder brother (B) is a MA
(C) whereas I am (D) only a BA

97. Choose the suitable phrasal verb for the blank in the sentence below:
I _____ my hopes when untimely rain threatened my crops.
(A) gave in (B) gave out
(C) gave up (D) gave off

98. Out of given alternatives, choose the word that is opposite in meaning to the word
Affluent
(A) Reluctant (B) Poor
(C) Clear (D) Enthusiastic

99. Fill in the blank with appropriate form of noun:
Don't blame yourself, it's not your …….
(A) misunderstanding (B) error
(C) slip (D) fault

100. Fill in the blank:
The instructor, along with the class _____ angry about the room change.

(A) are (B) have
(C) has (D) is

101. Choose the suitable word for the blank to make it a meaningful statement.
What you say is ..... my comprehension?
(A) before (B) beside
(C) behind (D) beyond

102. Fill in the blank with a suitable preposition:
If you want to avoid traffic, you need to leave ____ 7:30 am.
(A) until (B) by
(C) during (D) at

103. Choose the word the best expresses the meaning of the given idiom: 'To smell a rat'.
(A) To suspend something bad
(B) To misunderstand
(C) To detect bad smell
(D) To forsake

104. Out of the given alternatives, choose the word that best expresses the meaning of the word abridge.
(A) Judge (B) Release
(C) Shorten (D) Dissolve

105. 'A dog's breakfast' means
(A) breakfast cooked for a dog
(B) breakfast cooked by a dog
(C) something that has been done very badly
(D) None of the above

106. Change the speech:
She says, "I like going to the sea side".
(A) She says she likes going to the sea side
(B) She says I like going to the sea side
(C) She says that she liked going to the sea side
(D) She says she like going to the sea side

107. Arrange the following to form a correct sentence.
P : will normally be granted
Q : candidates should note
R : that no request for
S : change of centre
(A) SRQP (B) PRQS
(C) QSPR (D) QRSP

108. Rewrite the sentence after correcting the error.
She was one of the average student of the class.
(A) She was one of the average students of the class.
(B) She is one of the average student of the class.
(C) She was one among the average student of the class.
(D) She is an average students of the class.

109. Choose appropriate words to form a grammatically correct sentence.
The decoration of the new house, including the furniture and curtains .....
(A) is more pleasing (B) are more pleasing
(C) is most pleasing (D) are pleasing

110. Fill in the blank:
The President of the United States, accompanied by his advisors ..... enroute to Europe.
(A) were (B) are
(C) was (D) Both (A) & (C)

English:

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(A) were (B) are
(C) was (D) Both (A) & (C)

Computer:

111. All digital circuits can be realised by using only
(A) Exclusive OR gates (B) Half adders
(C) Multiplexers (D) OR gate

112. The Boolean function $a + (a.b)$ is equivalent to
(A) $a.b$ (B) $a + b$
(C) $a.\bar{b}$ (D) $\bar{a} + b$

113. Which of the following circuit is used as a memory device in computers?
(A) Flip-Flop (B) Rectifier
(C) Comparator (D) All of these

114. Convert the Hexadecimal number 4DF to its Octal
(A) 2333 (B) 2337
(C) 2773 (D) 2373

115. A tautology is a Boolean formula that is always true. Which of the Following is a tautology?
(A) x (B) $(x + \bar{x})y$
(C) $x + \bar{y} + \bar{x}$ (D) $(xy) + \bar{x}$

116. Acronym of EEPROM is
(A) Extended Erasable Programmable Memory
(B) Electrically Erasable Read Only Memory
(C) Electrically Erasable Programmable Read Only Memory
(D) Extended Erasable Page-Oriented Memory

117. For reproducing sound, a CD audio player uses a
(A) Quartz crystal (B) Titanium needle
(C) Barium ceramic (D) Laser beam

118. When we open an Internet site, we see WWW. What does WWW stand for?
(A) World Wide Word (B) World Wide Web
(C) World Wide Webinar (D) Word Widing Works

119. The answer of the operation $(10111)_2 \times (1110)_2$ in hex equivalent is
(A) 150 (B) 14C
(C) 142 (D) 13E

120. The minimum number of bits to represent a character from ASCII code set is
(A) 2 (B) 8
(C) 5 (D) 7

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Answer Key

1. A	13. A	25. D	37. C	49. C	61. D	73. A	85. D	97. C	109. C
2. A	14. D	26. D	38. A	50. B	62. D	74. B	86. C	98. B	110. C
3. D	15. B	27. C	39. B	51. C	63. A	75. C	87. B	99. D	111. B
4. A	16. D	28. C	40. A	52. B	64. C	76. B	88. B	100. D	112. B
5. B	17. D	29. C	41. A	53. A	65. B	77. C	89. A	101. D	113. A
6. D	18. D	30. D	42. D	54. A	66. C	78. D	90. C	102. B	114. B
7. A	19. B	31. B	43. B	55. C	67. D	79. C	91. D	103. A	115. C
8. C	20. C	32. A	44. A	56. D	68. D	80. A	92. C	104. C	116. C
9. D	21. A	33. D	45. B	57. B	69. C	81. D	93. D	105. C	117. D
10. D	22. D	34. B	46. C	58. B	70. A	82. D	94. A	106. A	118. B
11. C	23. C	35. D	47. B	59. D	71. B	83. C	95. B	107. D	119. C
12. D	24. A	36. C	48. B	60. D	72. A	84. A	96. B	108. A	120. D

Solution

1. (a) Given,

$$A=\begin{bmatrix}1&0&0\\0&1&1\\0&-2&4\end{bmatrix}$$

and $6A^{-1}=A^2+cA+dI$ ... (i)

Now, multiplying Eq. (i) by A, we get

$$6I=A^3+cA^2+dA$$

$$\Rightarrow A^3+cA^2+dA-6I=0 \quad \text{... (ii)}$$

Applying Cayley Hamilton rule

$$\begin{vmatrix}1-\lambda&0&0\\0&1-\lambda&1\\0&-2&4-\lambda\end{vmatrix}=0$$

$$\Rightarrow (1-\lambda)\{(1-\lambda)(4-\lambda)+2\}=0$$

$$\Rightarrow \lambda^3-6\lambda^2+11\lambda-6=0 \quad \text{... (iii)}$$

On comparing Eqs. (ii) and (iii), we get

$$c=-6 \text{ and } d=11$$

2. (a) Given, $\mathbf{a}\cdot\mathbf{b}=3$

and $(\mathbf{a}\times\mathbf{b})+\mathbf{c}=0$ ... (i)

Multiplying Eq. (i) by $\mathbf{b}$,

$$\mathbf{b}\cdot(\mathbf{a}\times\mathbf{b})+\mathbf{b}\cdot\mathbf{c}=0$$

$$\Rightarrow \mathbf{b}\cdot\mathbf{c}=0$$

Let $\mathbf{b}=x\hat{i}+y\hat{j}+z\hat{k}$, we get

$$y-z=3$$

and $x-y-z=0$

From these equations,

$$x=2z+3,\ y=z+3$$

$$\Rightarrow \mathbf{b}=(2z+3)\hat{i}+(z+3)\hat{j}+z\hat{k}$$

$\because (\mathbf{a}\times\mathbf{b})+\mathbf{c}=0$

So, $z=-2$

Thus, $\mathbf{b}=-\hat{i}+\hat{j}-2\hat{k}$

3. (d) As per the given condition,

$$\begin{aligned}A\cup B\cup C\cup D=&A+B+C+D-(A\cap B)-(A\cap C)\\&-(A\cap D)-(B\cap C)-(B\cap D)-(C\cap D)\\&+(A\cap B\cap C)+(A\cap B\cap D)+(A\cap C\cap D)\\&+(B\cap C\cap D)-(A\cap B\cap C\cap D)\\=&150+180+210+240-15-15-15\\&-15-15-15+3+3+3+3-0\\=&780-90+15-3=795-90-3\\=&705-3=702\end{aligned}$$

4. (a) Given, $ax+by+c=0$ passes through $(1,-2)$.

So, point will satisfy the given equation,

$$a\times1+b\times(-2)+c=0$$

$$\Rightarrow a-2b+c=0$$

$$\Rightarrow a+c=2b$$

Hence, a, b and c are in AP.

5. (b) Let the probability to show an even number be P.

As per the question,

Probability to show an odd number will be $3P$.

$$\therefore 3P+P=1$$

$$\Rightarrow 4P=1$$

$$\Rightarrow P=\frac{1}{4}$$

So, $3P=\frac{3}{4}$

Required probability $=\frac{3}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}=\frac{9}{16}+\frac{1}{16}=\frac{10}{16}=\frac{5}{8}$

6. (d) Given, $I_n=\int_0^{\pi/4}\tan^n\theta\,d\theta$

$$\Rightarrow I_n=\int_0^{\pi/4}\tan^{n-2}\theta\tan^2\theta\,d\theta$$

$$\Rightarrow I_n=\int_0^{\pi/4}\tan^{n-2}\theta(\sec^2\theta-1)\,d\theta$$

$[\because \tan^2\theta=(\sec^2\theta-1)]$

$$\Rightarrow I_n=\int_0^{\pi/4}\tan^{n-2}\theta\sec^2\theta-\int_0^{\pi/4}\tan^{n-2}\theta d\theta$$

$$\because I_n+I_{n-2}=\int_0^1 t^{n-2}\,dt=\left[\frac{t^{n-1}}{n-1}\right]_0^1 \quad [\because \text{let } \tan\theta=t]$$

$$\Rightarrow I_n+I_{n-2}=\frac{1}{n-1}$$

Now, putting $n=8$, we get

$$I_8+I_{8-2}=\frac{1}{8-1}$$

$$\Rightarrow I_8+I_6=\frac{1}{7}$$

7. (a) Given, area of triangle $=10\sqrt{3}$

$$\Rightarrow 10\sqrt{3}=\frac{1}{2}\times ab\times\sin\theta$$

$[\because \text{area of triangle}=\frac{1}{2}ab\sin\theta]$

$$\Rightarrow 10\sqrt{3}=\frac{1}{2}\times8\times5\sin\theta$$

$$\Rightarrow \sin\theta=\frac{10\sqrt{3}\times2}{8\times5}$$

$$\Rightarrow \sin\theta=\frac{2\sqrt{3}}{4}=\frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin\theta=\frac{\sqrt{3}}{2}\Rightarrow\theta=60^\circ \text{ or } 120^\circ$$

8. (c) A can hit the target 4 times in 5 attempts.

So, P (hit the target by A) $=\frac{4}{5}$

Similarly, P (hit the target by B) $=\frac{3}{4}$

and P (hit the target by C) $=\frac{2}{3}$

Now,

Required probability = P (target hit atleast two times)

$$=\frac{4}{5}\times\frac{3}{4}\times\frac{1}{3}+\frac{4}{5}\times\frac{1}{4}\times\frac{2}{3}+\frac{1}{5}\times\frac{3}{4}\times\frac{2}{3}+\frac{4}{5}\times\frac{3}{4}\times\frac{2}{3}$$

$$=\frac{1}{5}+\frac{2}{15}+\frac{1}{10}+\frac{2}{5}$$

$[\because A\bar{B}\bar{C}+A\bar{B}C+\bar{A}BC+ABC$ (atleast two times)$]$

$$=\frac{50}{60}=\frac{5}{6}$$

9. (*d*) Given,

$$I=\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \quad \text{... (i)}$$

$$\Rightarrow \quad I=\int_0^{\pi/2}\frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}}dx$$

$$\Rightarrow \quad I=\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \quad \text{... (ii)}$$

Now, adding Eqs. (i) and (ii), we get

$$2I=\int_0^{\pi/2}1dx \Rightarrow 2I=[x]_0^{\pi/2}$$

$$\Rightarrow \quad 2I=\frac{\pi}{2}\Rightarrow I=\frac{\pi}{4}$$

10. (*d*) $\begin{vmatrix}1+\omega & \omega^2 & -\omega\\ 1+\omega^2 & \omega & -\omega^2\\ \omega^2+\omega & \omega & -\omega^3\end{vmatrix}=\begin{vmatrix}-\omega^2 & \omega^2 & \omega\\ \omega & \omega & -\omega^2\\ 1 & \omega & -\omega^2\end{vmatrix}$

$[\because 1+\omega+\omega^2=0]$

$$=\omega^2\begin{vmatrix}-\omega & \omega & -1\\ -1 & 1 & -\omega\\ -1 & \omega & \omega^2\end{vmatrix}=\omega^2\begin{vmatrix}0 & \omega & -1\\ 0 & 1 & -1\\ \omega-1 & \omega & \omega^2\end{vmatrix}=\omega^2\times-3=-3\omega^2$$

11. (*c*) Given vectors are $2\hat{i}-3\hat{j}+0\hat{k}$,

$\hat{i}+\hat{j}-\hat{k}$ and $3\hat{i}+0\hat{j}-\hat{k}$

Volume of parallelopiped $=\begin{vmatrix}2 & -3 & 0\\ 1 & 1 & -1\\ 3 & 0 & -1\end{vmatrix}$

$$=2(-1)+3(2)=-2+6=4$$

12. (*d*) Let the term of GP be

$a, ar, ar^2, ar^3, ar^4, \ldots$

As per the question,

$$a=ar+ar^2\Rightarrow a(1)=a(r+r^2)\Rightarrow 1=r+r^2$$

$$\Rightarrow \quad 1-r-r^2=0 \Rightarrow r^2+r-1=0$$

Here, $a=1, b=1$ and $c=-1$

$$\therefore \quad r=\frac{-b\pm\sqrt{b^2-4ac}}{2\times a}=\frac{-1\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-1\pm\sqrt{(1)^2-4\times1\times(-1)}}{2\times1}=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1\pm\sqrt{5}}{2}$$

Since, r is positive.

Hence, $\quad r=\dfrac{-1+\sqrt{5}}{2}$

13. (*a*) Given, $f(x)=\tan^{-1}\left[\dfrac{\sin x}{1+\cos x}\right]$

$$\Rightarrow \quad f(x)=\tan^{-1}\left[\frac{2\sin x/2\cos x/2}{2\cos^2 x/2}\right]$$

$$\Rightarrow \quad f(x)=\tan^{-1}\left[\frac{\sin x/2}{\cos x/2}\right]$$

$$\Rightarrow \quad f(x)=\tan^{-1}[\tan x/2]\Rightarrow f(x)=x/2$$

$$\therefore \quad f'(x)=1/2$$

14. (*d*) Given, $\sin x+1=\cos x$

$$\Rightarrow \quad \cos x-\sin x=1$$

Dividing by $\sqrt{2}$ on both sides, we get

$$\frac{\cos x}{\sqrt{2}}-\frac{\sin x}{\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \cos\left(x+\frac{\pi}{4}\right)=\cos\frac{\pi}{4}$$

$$\therefore \quad x+\frac{\pi}{4}=2n\pi\pm\frac{\pi}{4}$$

On putting $n=0$,

$$x+\frac{\pi}{4}=0\pm\frac{\pi}{4}$$

$$\Rightarrow \quad x=0 \qquad [\because \text{taking} + \pi/4]$$

Now, putting $n=1$,

$$x+\frac{\pi}{4}=2\times1\times\pi\pm\frac{\pi}{4}\Rightarrow x=2\pi-\frac{\pi}{4}-\frac{\pi}{4}$$

$$\Rightarrow \quad x=\frac{3\pi}{2} \qquad [\because \text{taking} - \pi/4]$$

15. (*b*) As per the given condition,

T_n triangle can be formed by using the vertices of a regular polygon of n.

$$\therefore \quad {}^{n+1}C_3-{}^nC_3=21\Rightarrow {}^{7+1}C_3-{}^7C_3=21$$

Clearly, $\quad n=7$

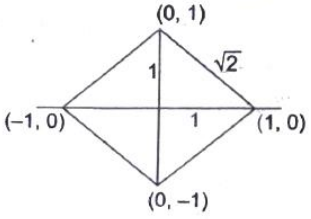
16. (*d*) Since, $\bar{x}_1<\bar{x}_2$

Also, $\bar{x}$ is the mean of combined distribution.

So, from options, option (d) is correct.

17. (*d*) Given, curve is $|x|+|y|=1$

The graph of curve $|x|+|y|=1$ is given below:



This is a square of side $\sqrt{2}$.

$$\therefore \quad \text{Area}=(\text{Side})^2=(\sqrt{2})^2$$

$$=2 \text{ sq units}$$

18. (d) Let the 2 degree polynomial be $f(x) = px^2 + qx + r$

$\therefore \quad f(1) = f(-1)$

$\Rightarrow \quad p + q + r = p - q + r \Rightarrow q = 0$

$\because \quad f'(x) = 2px$

So, $f'(a), f'(b), f'(c)$ will be $2pa, 2pb$ and $2pc$.

Hence, it is an AP.

19. (b) $\therefore$ Slope $= \frac{dy}{dx}$

$\Rightarrow \quad \frac{4}{2\sqrt{4x-3}} = \frac{2}{3}$

$\left[\because y = \sqrt{4x-3} - 1 \Rightarrow \frac{dy}{dx} = \frac{4}{2\sqrt{4x-3}}\right]$

$\Rightarrow \quad \sqrt{4x-3} = 3$

On squaring both sides, we get

$4x - 3 = 9$

$\Rightarrow \quad 4x = 12 \Rightarrow x = 3$

and $y = 2$

Hence, required point is (3, 2).

20. (c) Atal speak truth in 70% cases.

So, probability of Atal speak truth is $\frac{70}{100}$ i.e. $\frac{7}{10}$.

Similarly,

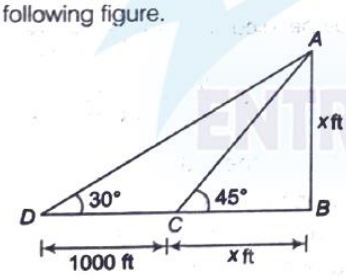
Probability of George speak truth is $\frac{40}{100}$ i.e. $\frac{4}{10}$.

As per the given condition, they are likely to contradict each other in stating the same fact only when one will speak truth.

$\therefore$ Required probability $= \frac{7}{10} \times \frac{4}{10} + \frac{3}{10} \times \frac{6}{10}$

$= \frac{28}{100} + \frac{18}{100} = \frac{28+18}{100} = \frac{46}{100} = \frac{23}{50}$

21. (a) As per the given information in the question, we can draw the following figure.



where,

$AB = x$ ft

$DC = 1000$ ft

$CB = x$ ft

In ΔABD,

$\frac{AB}{BD} = \tan\theta$

$\Rightarrow \quad \tan 30° = \frac{x}{x + 1000}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{x + 1000}$

$\Rightarrow \quad \sqrt{3}x = x + 1000$

$\Rightarrow \quad \sqrt{3}x - x = 1000$

$\Rightarrow \quad x = \frac{1000}{\sqrt{3} - 1}$

$\therefore \quad BD = 1000 + \frac{1000}{\sqrt{3}-1} = 500\sqrt{3}(\sqrt{3} + 1)$ ft

22. (d) $\because S_n = 216$

$\because \quad \frac{n}{2}[2a + (n-1)d] = 216$

where,

$T_n = 2n$

$a = n$

$2n = n + (n-1)d$

$n = (n-1)d$

and $d = \frac{n}{(n-1)}$

$\therefore \quad \frac{n}{2}\left[2n + (n-1) \times \frac{n}{(n-1)}\right] = 216$

$\Rightarrow \quad \frac{n}{2}[3n] = 216 \Rightarrow \frac{3n^2}{2} = 216$

$\Rightarrow \quad n^2 = \frac{216 \times 2}{3} = 72 \times 2$

$\Rightarrow \quad n^2 = 144 \Rightarrow n = 12$

$\therefore \quad d = \frac{12}{11}$

23. (c) Work done = Force × Displacement

$= (5\hat{i} + 3\hat{j} + 2\hat{k}) \times (2\hat{i} - 2\hat{j} + 10\hat{k})$

$[\because$ force $= 5\hat{i} + 3\hat{j} + 2\hat{k}$ and displacement $= 2\hat{i} - 2\hat{j} + 10\hat{k}]$

$= 10 - 6 + 20 = 24$

24. (a) $9^{\frac{1}{3}} \cdot 9^{\frac{1}{9}} \cdot 9^{\frac{1}{27}} \ldots$

$= 9^{\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots}$

$= 9^{\left(\frac{\frac{1}{3}}{1 - \frac{1}{3}}\right)} \quad \left[\because a + ar + ar^2 + \ldots \infty = \frac{a}{1-r}\right]$

$= 9^{\left(\frac{\frac{1}{3}}{\frac{3-1}{3}}\right)} = 9^{\frac{1}{3}/\frac{2}{3}} = 9^{\frac{1}{3} \times \frac{3}{2}} = 9^{1/2} = 3$

25. (d) As they can increase upto any value and decrease upto any level. So, there is no maxima and minima for any odd degree function.

26. (d) In word 'CORPORATION',

R repeated 2 times

O repeated 3 times

$\therefore$ Required probability

$= \frac{7!\,5!}{2!\,3!} = 7! \times 2 \times 5$

$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 2 \times 5 = 50400$

27. (c) Given, $\log_2 x = 10$

$\Rightarrow \quad x = 2^{10}$

$\therefore \quad \log_{2^{10}} y = 100$

$\Rightarrow \quad y = (2^{10})^{100} \Rightarrow y = 2^{1000}$

28. (c) Slope of line $2x - 3y = 7$ is $\frac{2}{3}$.

Since, required line is parallel to this line, so slope of the line will be $2/3$.

So, the mid-point of line segment joining the points $(1, 3)$ and $(1, -7)$ is $(1, -2)$.

$\therefore$ Required line is $y + 2 = \frac{2}{3}(x - 1)$

$\Rightarrow \quad 3y + 6 = 2x - 2$

$\Rightarrow \quad 2x - 3y - 8 = 0$

29. (c) $\because \quad (a+b)^2 - c^2 = ab$

$\Rightarrow \quad a^2 + b^2 + 2ab - c^2 = ab$

$\Rightarrow \quad a^2 + b^2 + ab - c^2 = 0$

$\Rightarrow \quad a^2 + b^2 - c^2 = -ab$

$\therefore \quad \cos c = \frac{a^2 + b^2 - c^2}{2ab}$

$\Rightarrow \quad \cos c = \frac{-ab}{2ab} = \frac{-1}{2} \Rightarrow c = 2\pi/3$

30. (d) $\because \quad 1000 = 10^3$

So, the numbers containing 1 is $10^3 - 9^3$.

31. (b) Since, lines are parallel.

So, distance between them is the diameter of the circle.

Lines are $3x - 4y + 4 = 0$

and $3x - 4y - \frac{7}{2} = 0$

$\therefore$ Distance between them

$$= \frac{4 - (-7/2)}{\sqrt{(3)^2 + (4)^2}} = \frac{15}{10} = \frac{3}{2}$$

So, radius $= \frac{3}{4}$

32. (a) If the sides of parallelogram are p and q. Then, their area is $|p \times q|$ = Magnitude of

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$= $ Magnitude of $-2\hat{i} - 14\hat{j} - 10\hat{k}$

$= \sqrt{300} = 10\sqrt{3}$

33. (d) Given,

$$\sin x + a\cos x = b \quad \ldots(i)$$

Let $\quad a\sin x - \cos x = p \quad \ldots(ii)$

Squaring and adding Eqs. (i) and (ii), we get

$\sin^2 x + a^2\cos^2 x + a^2\sin^2 x + \cos^2 x = b^2 + p^2$

$\Rightarrow \quad p^2 = 1 + a^2 - b^2$

Hence, $|p| = |a\sin x - \cos x| = \sqrt{1 + a^2 - b^2}$

34. (b) Given, $P(A \cup B) = 5/6$

$$P(A \cap B) = \frac{1}{3}$$

and $\quad P(B) = \frac{1}{2}$

Now, $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad \frac{5}{6} = P(A) + \frac{1}{2} - \frac{1}{3}$

$\Rightarrow \quad P(A) = \frac{4}{6} = \frac{2}{3}$

So, $\quad P(A) \cdot P(B) = P(A \cap B)$

$\therefore$ Events are independent.

35. (d) Since, vectors are coplanar.

$\therefore \quad \begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$

$\Rightarrow \quad \lambda = -4$

36. (c) Equation of base is $x + y = 2$.

Vertex is $(2, -1)$.

Length of perpendicular $= \left|\frac{2 - 1 - 2}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}}$

Side of the triangle is $\frac{h}{\sin B} = \frac{1/\sqrt{2}}{\sqrt{3}/2} = \sqrt{\frac{2}{3}}$

37. (c) Single digit number $= 3$

Two digit numbers are $3 \times 2 = 6$

Three digit numbers are $3 \times 2 \times 1 = 6$

Hence, total numbers are 15.

38. (a) Given, $x = a\cos t$

and $\quad y = b\sin t$

$\therefore \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-b}{a}\cot t$

Let $\quad u = \frac{dy}{dx} = \cot t$

$\therefore \quad \frac{d^2y}{dx^2} = \frac{du}{dx} = \frac{du}{dt} \times \frac{dt}{dx} = \left(\frac{b}{a}\text{cosec}^2 t\right) \times \left(\frac{-1}{a\sin t}\right)$

$= \frac{-b}{a^2\sin^3 t} = \frac{-b^4}{a^2 y^3}$

39. (b) Variance $= \bar{z}\,(x_i - \bar{x})^2\, p_i$

$\bar{x} = \bar{z}\, p_i x_i = 1\times 0.3 + 2\times 0.4 + 3\times 0.3 = 2$

$\therefore$ Variance $= (1-2)^2\times 0.3 + (2-2)^2\times 0.4 + (3-2)^2\times 0.3 = 0.6$

40. (a) $\because \tan A - \cot A = \dfrac{\sin^2 A - \cos^2 A}{\cos A \sin A}$

$= \dfrac{-2\cos 2A}{\sin 2A} = -2\cot 2A$

Adding and subtracting $\cot\theta$ in the equation, we get

$\tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta$

$= \tan\theta - \cot\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta + \cot\theta$

$= -2\cot 2\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta + \cot\theta$

41. (a) First term multiplied by 7 in between 200 and 400, is 203.

$\therefore$ $203 + 210 + \ldots + 399$ [$\because$ last term = 399]

There are $\dfrac{399-203}{7} + 1 = 29$ terms in the series.

$\therefore$ Sum of 29 terms $= \dfrac{(203+399)}{2}\times 29 = 8729$

42. (d) $\lim\limits_{x\to 0} \dfrac{\tan x - x}{x^2 \tan x}$

$= \lim\limits_{x\to 0}\left[\dfrac{\sec^2 x - 1}{x^2\sec^2 x + 2x\tan x}\right]$

Applying L' Hospital's rule,

$= \lim\limits_{x\to 0}\left[\dfrac{1-\cos^2 x}{x^2 + 2x\sin x\cos x}\right]$

$= \lim\limits_{x\to 0}\left[\dfrac{\sin^2 x}{x^2 + 2x\sin x\cos x}\right]$

$= \lim\limits_{x\to 0}\left[\dfrac{\sin^2 x / x^2}{1 + \dfrac{2\sin x}{2}\cdot\cos x}\right] = \dfrac{1}{3}$

43. (b) Sum is a prime number i.e. the sum of the dice outcomes can be 2, 3, 5, 7, 11.

These sums are shown in following pairs

(1, 1), (1, 2), (2, 3), (3, 4), (5, 6)
(5, 2), (4, 1), (2, 1), (4, 3), (6, 5)
(2, 5), (1, 4), (3, 2), (1, 6), (6, 1)

$\therefore$ Required probability $= \dfrac{15}{36} = \dfrac{5}{12}$ [2 dice $= 6\times 6 = 36$]

44. (a) Since, centre lies on the line $x + 2y + 3 = 0$.

Let the centre be $(-2a-3, a)$, then its distance from the given two points $(-1, 1)$ and $(2, 1)$ is

$(-2a-3+1)^2 + (a-1)^2 = (-2a-3-2)^2$

$+ (a-1)^2$

$(-2a-2)^2 = (-2a-5)^2$

$\Rightarrow$ $a = -\dfrac{7}{4}$

Thus, the centre is $\left(\dfrac{1}{2}, \dfrac{7}{4}\right)$.

Radius is $\sqrt{\dfrac{157}{16}}$.

So, the equation of the circle is

$$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{7}{4}\right)^2 = \frac{157}{16}$$

$\Rightarrow$ $2x^2 + 2y^2 - 2x + 7y - 13 = 0$

45. (b) The two adjacent sides of the triangle are represented by $(\mathbf{b}-\mathbf{a})$ and $(\mathbf{c}-\mathbf{a})$, thus its area $= \dfrac{1}{2}|(\mathbf{b}-\mathbf{a})\times(\mathbf{c}-\mathbf{a})|$

$= \dfrac{1}{2}|\mathbf{b}\times\mathbf{c} - \mathbf{b}\times\mathbf{a} - \mathbf{a}\times\mathbf{c}| = \dfrac{1}{2}|\mathbf{b}\times\mathbf{c} + \mathbf{a}\times\mathbf{b} + \mathbf{c}\times\mathbf{a}|$

46. (c) $\therefore$ Standard result regarding distance between two focal $PS + PS' = 2a$

47. (b) Given, $\sin x + \sin^2 x = 1$

$\Rightarrow$ $\sin x = 1 - \sin^2 x \Rightarrow \sin x = \cos^2 x$

$\therefore$ $\cos^2 x = \sin x$

So, given equation $\cos^2 x + \cos^4 x$

can be written as $\cos^2 x + \cos^{2\times 2} x$ i.e. $\sin x + \sin^2 x$

where, $\sin x + \sin^2 x = 1$

48. (b) Suppose probability of success is $2p$ and failure is p.

Then, $2p + p = 1 \Rightarrow p = \dfrac{1}{3}$

Hence, probability of getting success is $\dfrac{2}{3}$ and failure is $\dfrac{1}{3}$.

$\therefore$ Required probability $= P$ (atleast 4 success)

$= {}^6C_4\,(2/3)^4\left(\dfrac{1}{3}\right)^2 + {}^6C_5\,(2/3)^5\,(1/3)^1 + {}^6C_6\,(2/3)^6$

$= \dfrac{15\times 16 + 6\times 32 + 64}{3^6} = \dfrac{16(15+12+4)}{3^6} = \dfrac{496}{729}$

49. (c) Given, series $-1^2 + 2^2 - 3^2 + 4^2 + \ldots$

$= \{(-1)^2 + (2)^2\} + \{(-3)^2 + (4)^2\} + \ldots + \{(-19)^2 + (20)^2\}$

$= 3 + 7 + 11 + \ldots + 39$

Required sum (for 10 terms)

$= \left(\dfrac{3+39}{2}\right)\times 10 = 210$

50. (b) Given, $\tan\alpha = \dfrac{m}{m+1}$ and $\beta = \dfrac{1}{2m+1}$

$\therefore$ $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha - \tan\beta}$

$= \dfrac{\dfrac{m}{m+1} + \dfrac{1}{2m+1}}{1 - \left(\dfrac{m}{m+1}\right)\left(\dfrac{1}{2m+1}\right)} = \dfrac{2m^2+2m+1}{2m^2+2m+1} = 1$

$\Rightarrow$ $\tan(\alpha+\beta) = 1$

$\Rightarrow$ $\alpha + \beta = \pi/4$

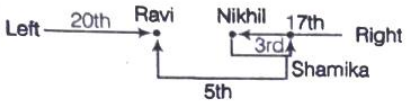
51. (c) Let the length of the train be x m.

Then, $\dfrac{x+162}{18} = \dfrac{x+120}{15}$

$\Rightarrow$ $\dfrac{x+162}{6} = \dfrac{x+120}{5}$

$\Rightarrow$ $6x + 720 = 5x + 810 \Rightarrow x = 90$ m

52. (b)

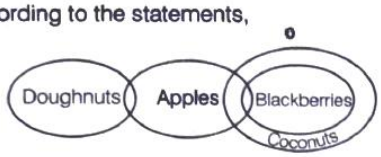


∴ Ravi's position from the left = 20th

and Ravi's position from the right = 17 + 2 = 19th

∴ Number of children in the row = (20 + 19) − 1 = 38

53. (a) According to the statements,



Hence, some blackberries are doughnuts, is definitely, false.

Solutions [Q. Nos. 54 to 56]

Reena's mother-in-law → head of the family → Lawyer

Rakesh → Married → Teacher

Mukesh → Married → Teacher

Reena → Rakesh's wife → Lawyer

Mukesh's wife → Doctor

Ajay → Mukesh's son

Ajay's grandfather → Unknown profession

54. (a) Rakesh (Teacher) and Reena (Lawyer) is definitely a couple.

55. (c) Rakesh's wife's profession is Lawyer.

56. (d) We cannot determine the Ajay's grandfather occupation.

57. (b) The pattern of series is as follows:

$$A \xrightarrow{(B)} CD \xrightarrow{(E,F)} GHI \xrightarrow{(J,K,L)} MNOP \xrightarrow{(Q,R,S,T)} UVWXY$$

58. (b) As, $F \xrightarrow{+1} G$, $R \xrightarrow{+2} T$, $I \xrightarrow{+3} L$, $E \xrightarrow{+4} I$, $N \xrightarrow{+5} S$, $D \xrightarrow{+6} J$

Similarly, $G \xrightarrow{+1} H$, $U \xrightarrow{+2} W$, $A \xrightarrow{+3} D$, $R \xrightarrow{+4} V$, $D \xrightarrow{+5} I$

59. (d) Given, Alan's age = 34 yr

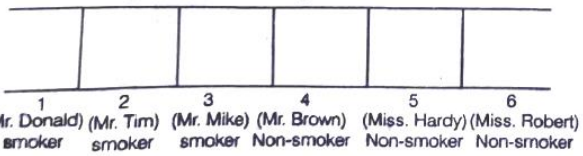
Carl's age = 34 + 2 = 36 yr

Bob's age = 36 − 1 = 35 yr

Dave's age = 35 + 2 = 37 yr

Hence, Dave is the oldest among them.

Solutions [Q. Nos. 60 to 62]



60. (d) Mr. Donald's office is farthest from Mr. Brown.

61. (d) The three smoker employers of the offices should be seated in 1, 2 and 3.

62. (d) The ideal office for Mr. Mike would be 3.

63. (a) Doctor's previous time to see the patients

= 1: 40 pm − 3 h 30 m = 10:10 am

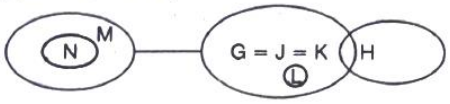
So, the information given to the compounder by the doctor

= 10 : 10 am + 1 : 20 = 11: 30 am

64. (c) Series is increasing in three consecutive terms like

42 40 38; 35 33 31; 28 26 24

Solutions [Q. Nos. 65 to 66]



65. (b) If no P's are K's, then if any P is a G, it is a J, must be true.

66. (c) Some H's are both M's and G's is inconsistent with one or more of the conditions.

67. (d) According to the question,

Shyam > Pradeep = Anurag = Suresh > (Anand, Praveen)

Hence, Shyam is tallest among them.

68. (d) Suppose,

Rajeev's age = 0 (Born)

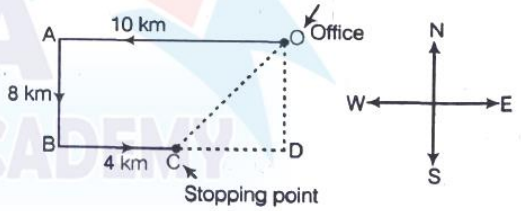
Then, Rajeev's brother's age = 6 yr

Rajeev's father's age = 6 + 32 = 38 yr

Rajeev's mother's age = 38 − 3 = 35 yr

Rajeev's sister's age = 35 − 25 = 10 yr

69. (c) Dhoni's walking directions are as follows:



∴ OD = AB = 8 km

⇒ CD = BD(AO) − BC

= (10 − 4) km

= 6 km

∴ Required distance (OC) $= \sqrt{OD^2 + CD^2}$

$= \sqrt{8^2 + 6^2} = \sqrt{64 + 36}$

$= \sqrt{100} = 10$ km

70. (a) By option, number 58 follows the statements rule.

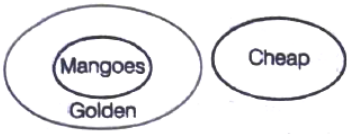
$58 = \underline{3} \times 19 + \boxed{1}$

$58 = \underline{4} \times 14 + \boxed{2}$

$58 = \underline{5} \times 11 + \boxed{3}$

$58 = \underline{6} \times 9 + \boxed{4}$

71. (*b*) According to the statements, Venn diagram is as follows:



Conclusion
I. ✗
II. ✓

Hence, only conclusion II follows.

Solutions [Q. Nos. 72 to 73]
According to the question.

$$A = 2B \quad \ldots (i)$$
$$B = 4.5\,C \quad \ldots (ii)$$
$$C = \frac{1}{2}D \quad \ldots (iii)$$
$$D = \frac{1}{2}E \quad \ldots (iv)$$
$$A > E > C \quad \ldots (v)$$

Combining all the equations,
$A > B > E > D > C$

72. (*a*) A is the heaviest article in weight.

73. (*a*) A, B, E, D, C

Solutions [Q. Nos. 74 to 76]
A (only ON) → B (ON) ...(i)
A (ON) + B (ON) → C (ON) ...(ii)
A (ON) + B (ON) + C (ON) → C (OFF) ...(iii)
Except,
ON → OFF and OFF → ON

74. (*b*) If switches A and B are ON and C is OFF, their changed settings will be A ON, B ON and C ON.

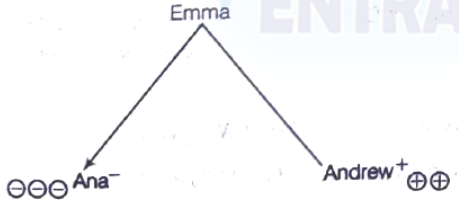
75. (*c*) If only B is ON, the changed setting will be A ON, B OFF and C ON.

76. (*b*) If only B is ON in the changed setting, the original setting could have been A ON, B OFF and C ON.

77. (*c*) ∴Fourth date after 21st $= 21 + 4 = 25$

Then, 3 ↑ 10 ↑ 17 ↑ 24 ↑
1st Friday 2nd Friday 3rd Friday 4th Friday
∴Required day (25th date) = Friday + 1 = Saturday

78. (*d*)



∴ Number of children = $\ominus + \ominus + \ominus + Ana^- + Andrew + \oplus + \oplus$
$= 7$

Solutions [Q. Nos. 79 to 81]
Anu > Cini ...(i)
Binu > Eenu ...(ii)
Dany > Anu ...(iii)
Eenu > Anu ...(iv)
Combining all the equations,
(Dany, Binu) > Eenu > Anu > Cini

79. (*c*) Dany or Binu is the tallest among them.

80. (*a*) Cini is the shortest among them.

81. (*d*) Dany is the tallest in the group, which would help to logically order the persons according to their heights.

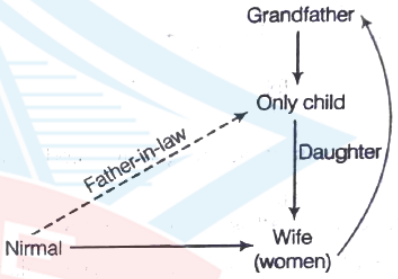
82. (*d*) In Ist 100 m race, distane covered by Karan = 100 m and distance covered by Arjun = 90 m
In 2nd 100 m race, distance covered by Karan = 110 m and distance covered by Arjun = ?
100 m of Karan = 90 m of Arjun

$$110 \text{ m of Karan} = \frac{90 \times 110}{100} = 99\text{m of Arjun}$$

∴Karan beats Arjun by 1 m.

83. (*c*) India (+ 2) → Australia (– 2)
West Indies (+ 2) → India (– 2)
[+ → win, – → loss]
Australia (+ 2) → West Indies (– 2)
India (+ 2) → New Zealand (– 2)
West Indies (+ 2) → New Zealand (– 2)
Hence, New Zealand Country has lost 4 matches most number of times.

84. (*a*)



So, women is related as wife of the Nirmal.

Solutions [Q. Nos. 85 to 87]

Condition I	Condition II	Condition III	Condition IV
6 – C	6 –	6 ... B	6 ... B
5 –	5 – C	5 ... D	5 ... B,D
4 – A	4 –	4 – C	4 ... D
3 ... B	3 – A	3 –	3 – C
2 ... B, D	2 ... B	2 – A	2 –
1 – D	1 ... D	1 –	1 – A

85. (*d*) According to condition IV, if A is standing on step 1, then D is standing one step higher than C.

86. (*c*) According to conditions I and II, if D is standing on step, 1 then A could be standing on either step 3 or 4 only.

87. (*b*) According to conditions I, III and IV, if there are two steps in between the steps A and D, then A may be standing in 4, 2 and 1 steps. So, in respect of option, we say that A must be standing on step 4.

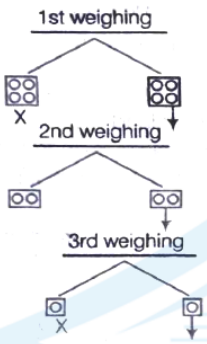
88. (*b*) $A \times B = A = B$

$A - B \Rightarrow A > B$
$A + B \Rightarrow A < B$

Then, Sachin * Mohan – Ravi
⇒ Sachin = Mohan > Ravi
Hence, Ravi is youngest among them.

89. (a) Total movement of hour hand = 20° + 20° = 40°
So, hour hand covers $\frac{1^\circ}{2}$ in 1 min.
Also, hour hand covers 40° in 80 min.
Time duration to go tennis court and returning in (10 + 10) min
= 20 min
∴ Total spent time at the tennis court = (80 – 20) min
= 60 min

90. (c)



Hence, three times of weighing is required to identify the heavier ball.

91. (d) 'Someone who died' is the correct phrase.

92. (c) Easy choice from among the given alternatives is Gold: Bangle.

93. (d) *Angry with somebody* is used.

94. (a) The Earth revolves round the Sun.

95. (b) *A close shave* means 'a narrow escape'.

96. (b) MA begins with a vowel sound, so 'an' will be used.

97. (c) *To gave up something* is used.

98. (b) Affluent means 'having a lot of money'.

99. (d) 'Fault' is the appropriate noun here.

100. (d) Singular verb is required here.

101. (d) 'beyond any comprehension' is the correct phrase.

102. (b) 'By 7:30 am' means not later than 7:30 am.

103. (a) *To smell a rat* means 'to suspect something bad'.

104. (c) *Abridge* means to shorten a play/movie etc by leaving parts or sections out.

105. (c) *A dog's breakfast* means 'something that has been done very badly'.

106. (a) She says that she likes going to the sea side.

107. (d) Candidates should note that no request for change of centre will normally be granted.

108. (a) She was one of the average students of the class.

109. (c) *is most pleasing* is correct.

110. (c) *was* is appropriate.

111. (b) All digital circuits can be realised by using only half adders. A half adder consists of an XOR gate and an AND gate.

112. (b) The Boolean function $a + (a' \cdot b)$ is solved as

$$\begin{aligned} & a + (a' \cdot b) \\ &= (a' + a') \cdot (a + b) && \text{[using distributive law]} \\ &= 1 \cdot (a + b) && [\because a + a' = 1] \\ &= a + b \end{aligned}$$

113. (a) The flip-flop circuit is used as a memory device in computers.

114. (b) Conversion of hexadecimal 4DF to its octal as

$$\overset{4}{0100}\ \overset{D}{1101}\ \overset{F}{1111} = (010011011111)_{16}$$

Now, $(010011011111)_{16}$ into octal number is

$$\frac{010}{2}\ \frac{011}{3}\ \frac{011}{3}\ \frac{111}{7} = (2337)_8$$

115. (c) As we known a tautology is a Boolean formula that is always true. From the giving options, we solve all through the truth table.
Here, $x + \bar{y} + \bar{x}$ is satisfying the condition of tautology.

x	y	$\bar{x}$	$\bar{y}$	$x + \bar{y} + \bar{x}$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

The output of $x + \bar{y} + \bar{x}$ is true.

116. (c) EEPROM stands for Electrically Erasable Programmable Read Only Memory. It is a type of a non-volatile memory used in computers and other electronic devices.

117. (d) Compact Disc (CD) is a disc which contains digital data that is scanned by a laser beam for the reproduction of recorded sound.

118. (b) WWW stands for World Wide Web.

119. (c) The multiplication of $(10111)_2 \times (1110)_2$ is

$(10111)_2 \times (1110)_2 = (000101000010)_2$
Now, the result is converted into hexadecimal as

$$\underset{1}{0001}\ \underset{4}{0100}\ \underset{2}{0010}$$

So, the answer of the operation $(1011)_2 \times (1110)_2$ in hex equivalent is $(142)_{16}$.

120. (d) The minimum number of bits to represent a character from ASCII code set is 7.

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